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Pasadena, California

Geostatistical interpolation of AIRS mid-tropospheric CO2

Hai Nguyen[1] Edward Olsen[1] Noel Cressie[2] Amy Braverman[1]

[1] Jet Propulsion Laboratory,
California Institute of Technology

[2] Department of Statistics,
The Ohio State University

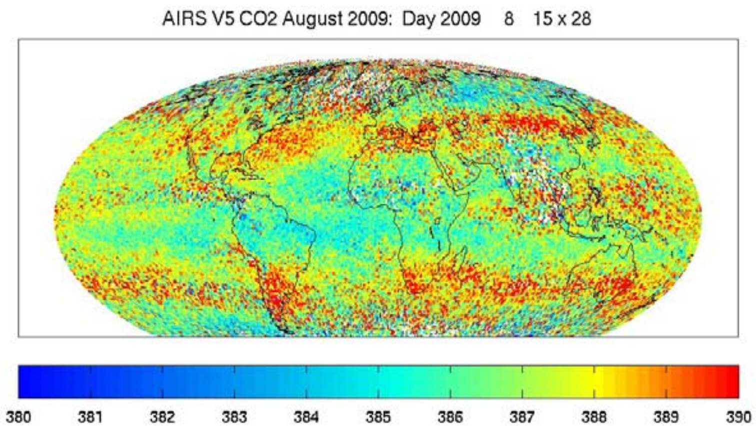
The work described here is generously supported by NASA's Earth Science Technology Office through its Advanced Information Systems Technology Program.

Outline

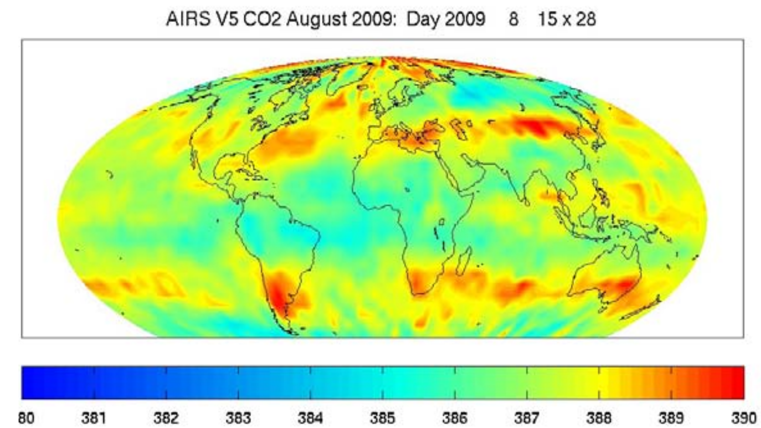
- Introduction
- Fixed Rank Kriging (FRK)
- 9-year AIRS CO2 movies
- Conclusions

Introduction

- Motivation: spatial interpolation of remote sensing data.



AIRS CO2, August 2009; gridded



AIRS CO2, August 2009;
moving window average, 5 degree tophat

- Spatial interpolation is a necessary fact of life in analyzing remote sensing CO2.

- Examples include simple "gridding", linear interpolation, and moving window averaging, etc.

- Not all spatial interpolation methods provide measures of uncertainty. A spatial statistical model makes this possible.

Traditional Kriging

Let $\mathbf{Z}$ be an N-dimensional vector of observed AIRS values at locations $(s_1, s_2, \ldots, s_N)$.

We estimate the AIRS value at location s_0 with the following form,

$$\hat{Y}(s_0) = \mathbf{a}(s_0)' \mathbf{Z},$$

where $\mathbf{a}(s_0)$ is an N-dimensional vector of kriging coefficients. We wish to find the vector $\mathbf{a}(s_0)$ that minimizes the expected squared error,

$$E([Y(s) - \mathbf{a}(s_0)' \mathbf{Z}]^2).$$

Traditional Kriging

Solving for the kriging coefficient vector using matrix derivatives, we get,

$$\mathbf{a}(s_0) = \boldsymbol{\Sigma}^{-1}\, \mathbf{c}(s_0)',$$

where

$\boldsymbol{\Sigma}^{-1} = \mathrm{Var}(\mathbf{Z})$ is an NxN matrix,
$\mathbf{c}(s_0)' = \mathrm{Cov}(\mathbf{Z}, Y(s_0))$ is an Nx1 vector.

- To compute the kriging coefficients, $\mathbf{a}(s)$, we need to compute the inverse of the (N x N) covariance matrix $\boldsymbol{\Sigma} = \mathrm{var}(\mathbf{Z})$.

Fixed Rank Kriging

Fixed ranked kriging (FRK) models the covariance structure with the following form,

$$\text{cov}(\ Y(s_1),Y(s_2)\ ) = \mathbf{S}(s_1)'\ \mathbf{K}\ \mathbf{S}(s_2),$$

which leads to the following form for the covariance matrix **Σ,**

$$\mathbf{\Sigma} = \mathbf{S}'\ \mathbf{K}\ \mathbf{S} + \mathbf{D},$$

Because of this special form, **Σ** can be inverted quickly using the Sherman-Morrison-Woodbury formula,

$$\mathbf{\Sigma}^{-1} = \mathbf{D}^{-1} - \mathbf{D}^{-1}\,\mathbf{S}'\,(\ \mathbf{K}^{-1} + \mathbf{S}\,\mathbf{D}^{-1}\,\mathbf{S}'\ )^{-1}\,\mathbf{S}\,\mathbf{D}^{-1}.$$

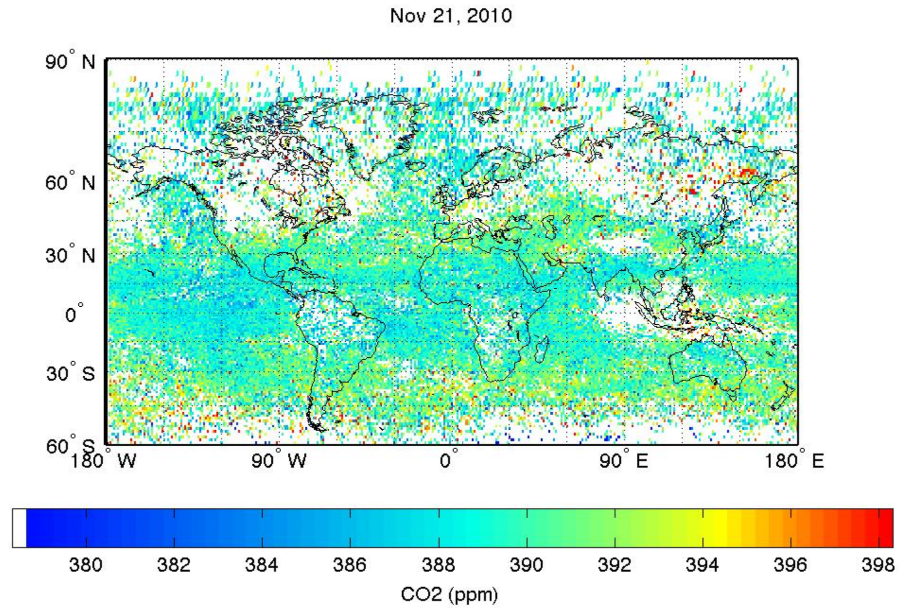
- **S**(s) is an r-dimensional spatial basis expansion of s,
- **K** is an (r x r) matrix,
- **S** is an (r x N) matrix of **S**(•) evaluated at all observation location,
- **D** is an (N x N) diagonal matrix of measurement-error variance.

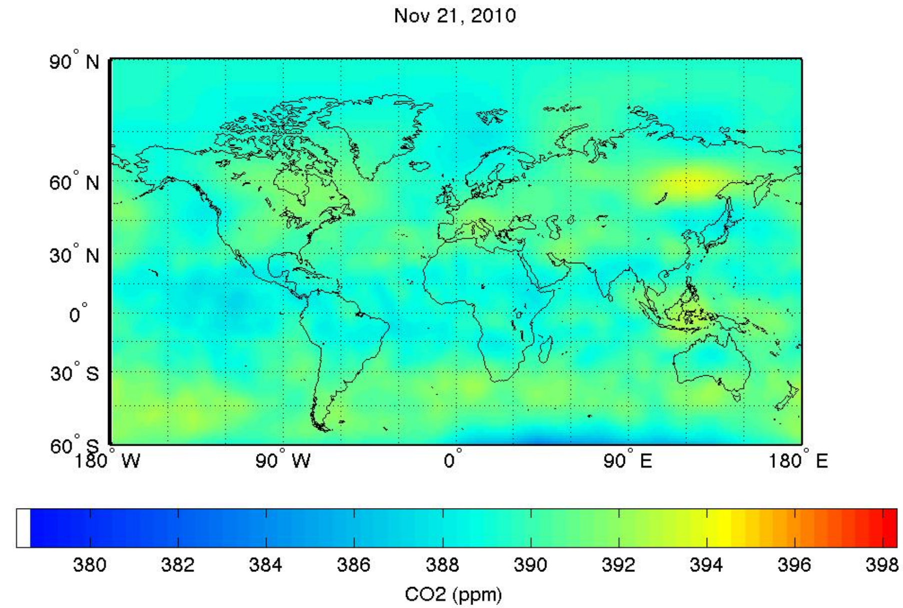
Fixed Rank Kriging

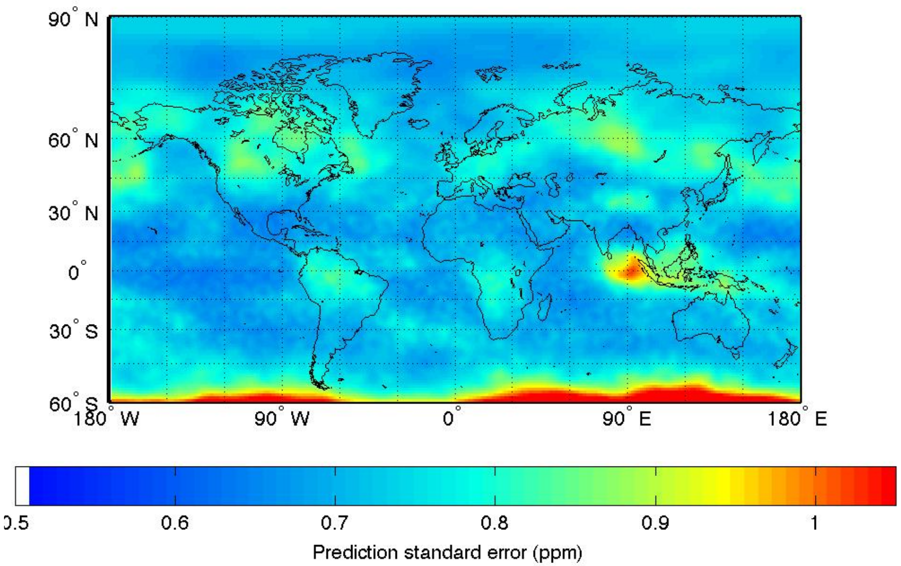
FRK can handle massive amount of data with low computational burden.

- FRK properly accounts for the spatial dependence between observations,
- It produces estimates of prediction error, which allow for hypothesis testing,
- It is well-suited for producing Level 3 data,
- In this presentation, we apply FRK to global AIRS CO2 record (September 2002 – January 2012), using 9-day moving window.

FRK Outputs







Sample FRK input and outputs for 1 time period: November 21, 2010.

Top left: raw input data
Top right: FRK CO_2 estimates
Bottom left: FRK error estimates

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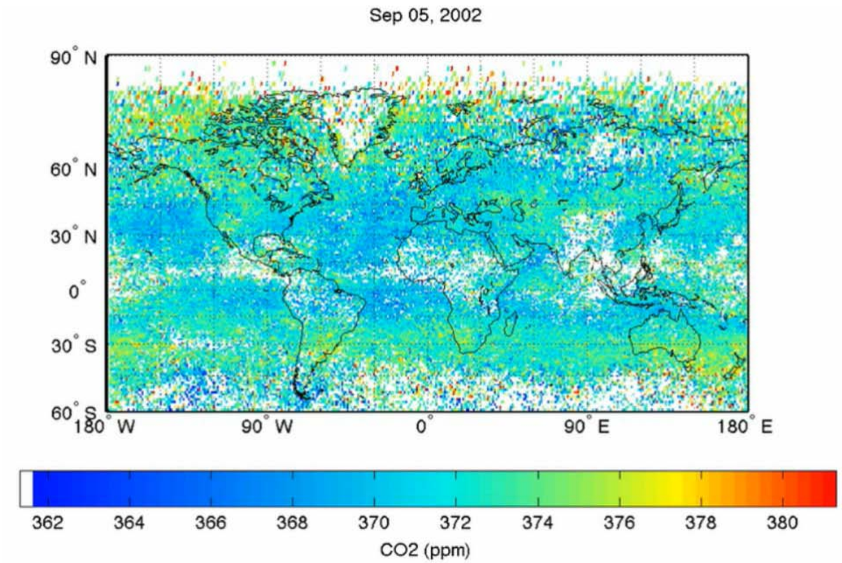
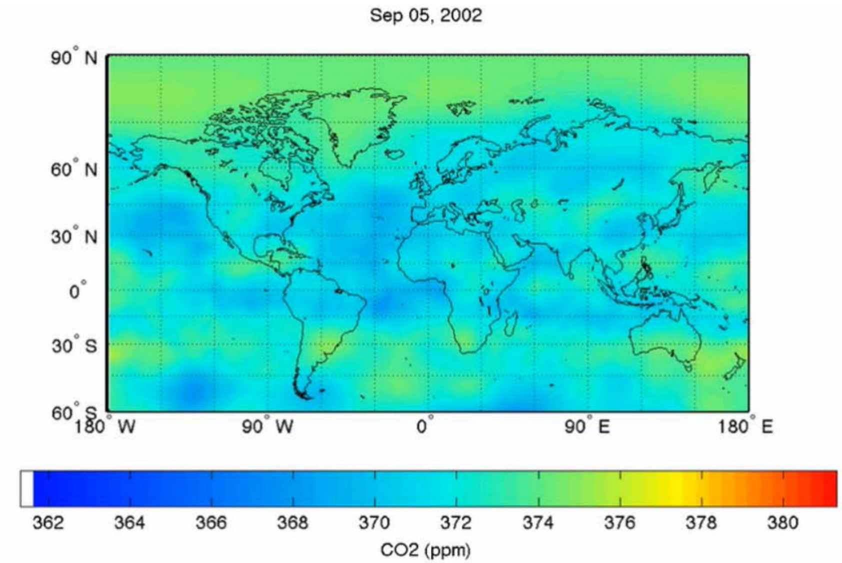
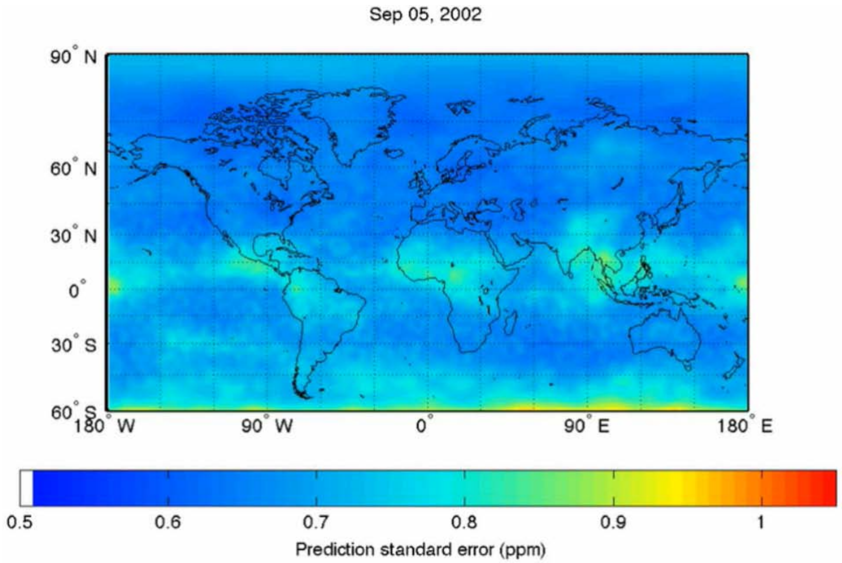
FRK movies







Top left: AIRS raw CO2 map
Top right: AIRS FRK map
Bottom left: AIRS FRK error map

FRK movies

Some notes:

- The higher uncertainties in the Bay of Bengal, Africa, and South America are due to lower yields.

- The increased uncertainty in the northern latitudes in 2011 may be due to loss of L2 yield as a result of the steady degradation of AMSU channel 5.

- The belt of elevated CO_2 in the Southern Hemisphere is an annually recurring feature.

- This cylindrical projection distorts the Arctic region, leading to an exaggerated perception of data sparseness in the region due to the fact that the resolution of the output is $1° \times 1°$.

Conclusions

- Smoothed maps of AIRS CO2 does well in highlighting movement and evolution of mid-tropospheric CO2 throughout the 9-year timespan.

- FRK can provide a new type of Level 3 product with the following properties:
    - No spatial gap and a temporal resolution of a few days.
    - Outputs may be customized to match any desired spatial resolution.
    - The technique may be generalized to work in the third dimension (altitude) and employed for other physical parameters.

Appendix A: projection distortion