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Errors Arising from Estimating Level Composition Products from Layer-Mean Products

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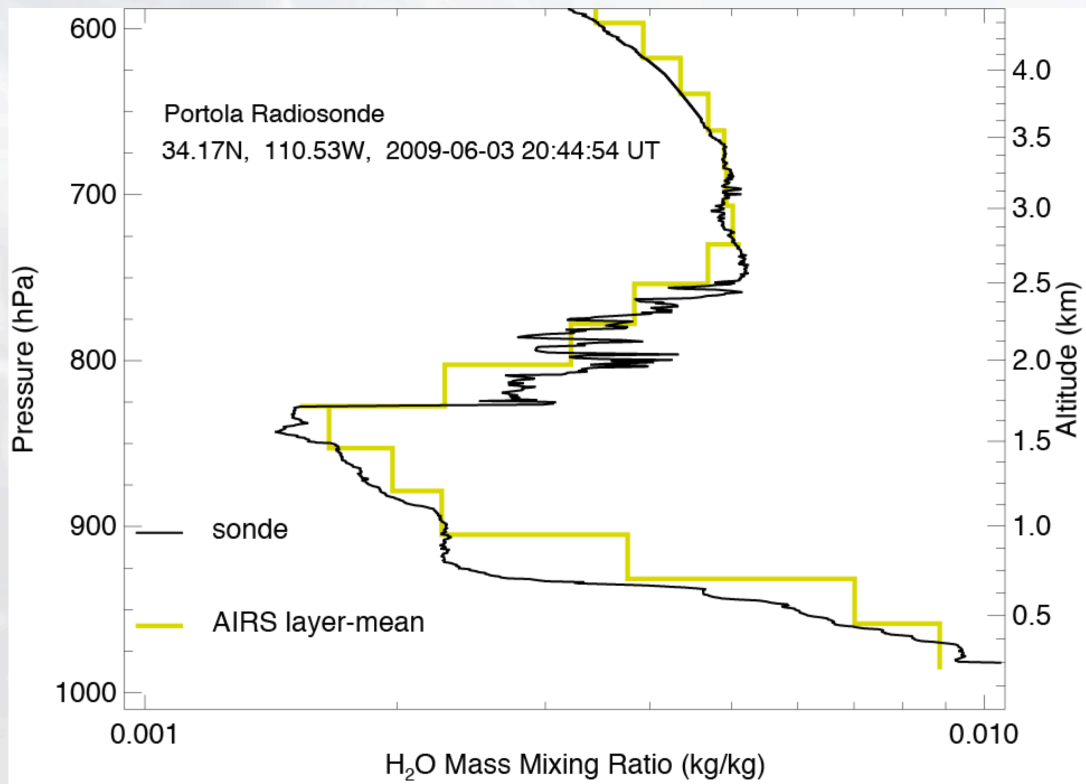
Introduction

- The inverse problem

Vector of layer-mean mixing ratios → Continuous profile of mixing ratios

- How do we reconstruct the original profile



The real problem:

How to recast to a more product.

- Input to rta's
- Calculate lapse rates, and ePV.

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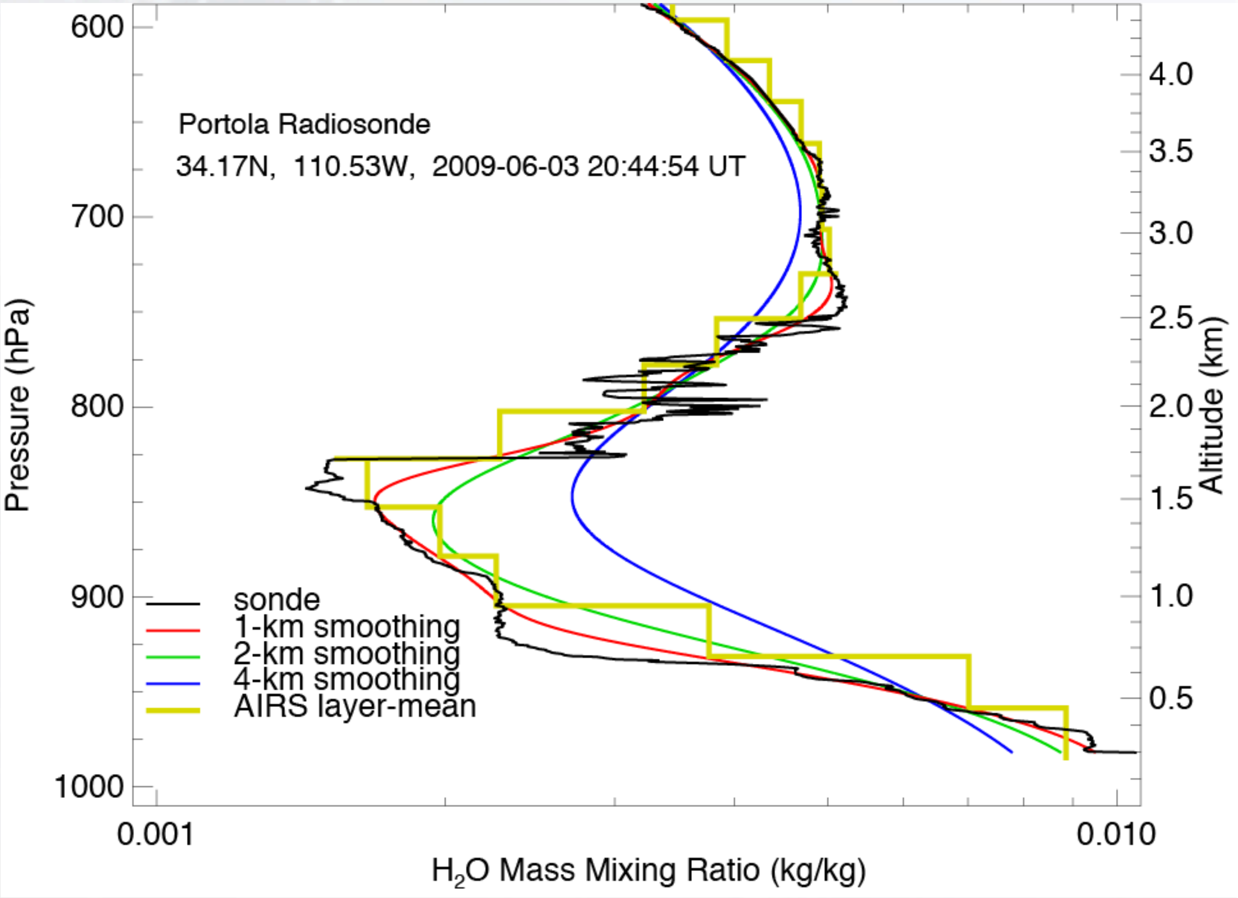
Expected Resolution

- Radiosonde
  - 1-second sampling
  - Converted to 5-m vertical resolution
  - Gaussian-smoothed

- AIRS/CrIMSS RTA layering supports 1-2 km vertical resolution in the PBL



Approaches

- Mean-value Technique (MVT)
  - Locate layer-mean value
  - Fit curve
  - Interpolate to levels
- Generalized Mean-value Technique
  - Evaluate space to level integrated amount
  - Fit curve to integrated amount
  - Interpolate to new layering and evaluate new mean values
  - Apply MVT technique to new layers
- Bayesian curve fitting
  - Solve for curve fitting layer-means
  - Add prior knowledge about profiles*

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Mean Value Technique

- Mean (wet) mixing ratio within a layer
  - Layer amount (kg/m²) or (molecules/cm²) $\bar{X}_i = \frac{cd_i}{\Delta m_i}$
  - Layer mass $\Delta m_i = \frac{\Delta P}{g}$ or more exactly: cd_i

$$\Delta m_i = \int_{P_i}^{P_{i-1}} \frac{dP}{g(z,\theta,\lambda)}$$

$$z(P) = \int_{P_{\text{Surf}}}^{P} \frac{RT}{g(z,\theta,\lambda)} d\ln P$$

- The mean value technique (mvt)
  - Assumes the mean value occurs at a midpoint of the layer

Arithmetic	Geometric	Logarithmic
$\bar{P}_i = \frac{1}{2}(P_i + P_{i-1})$	$\bar{P}_i = \sqrt{P_i P_{i-1}}$	$\bar{P}_i = \frac{P_i - P_{i-1}}{\log\left(\frac{P_i}{P_{i-1}}\right)}$

  - Interpolate (extrapolate) to the levels

$$X(P) = \mathcal{I}(\bar{P}_i \ldots \bar{P}_n; \bar{X}_1 \ldots \bar{X}_n)(P)$$

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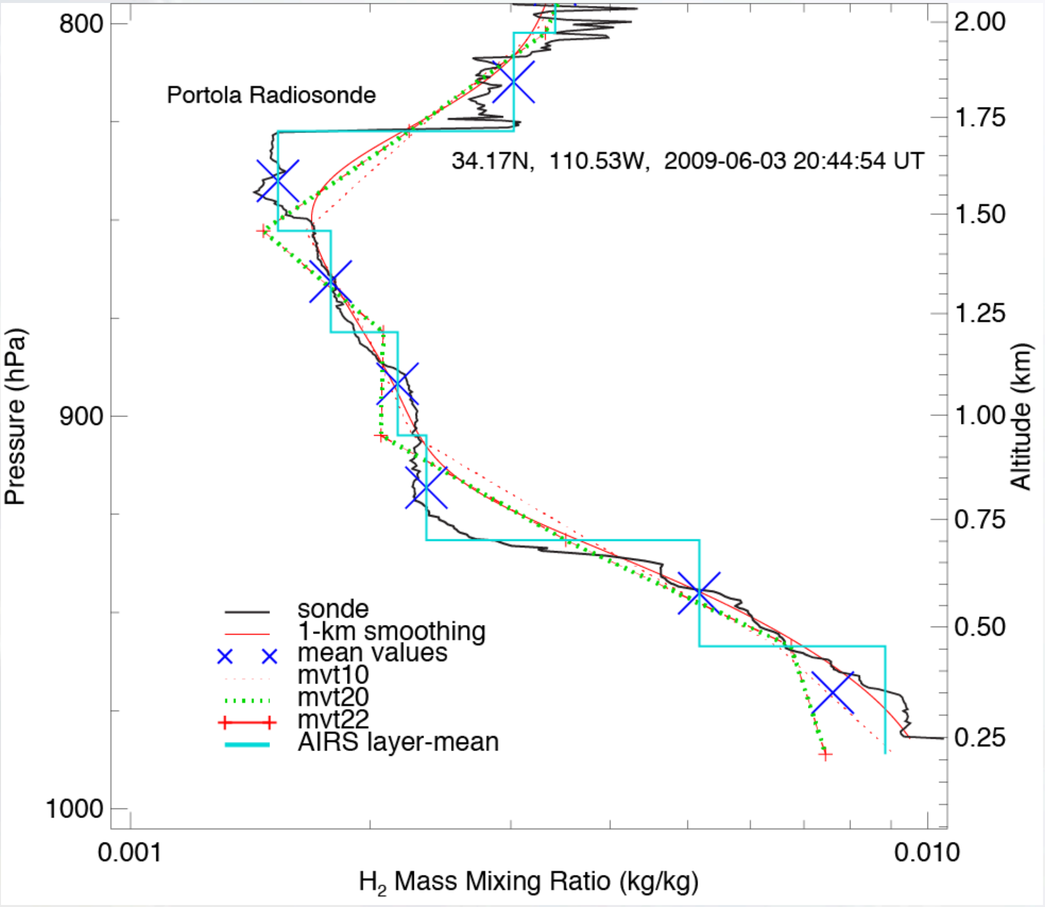
Example Mean Value Technique

- Arithmetic means
  - Linear interpolation (mvt10)
  - Cubic-spline interpolation (mvt20)
  - Blue x's are arithmetic means
- Logarithmic means
  - Cubic-spline interpolation (mvt22)
- Error Sources
  - Assumed position
  - Functional form of interpolating function



Generalized Mean Value Technique

- Interpolation and differencing on integrated level to level amount $\mathrm{tcd}_i = \sum_{j=0}^{i} \mathrm{cd}_j$
- Interpolate and difference tcd_i to arbitrary grid, $P(z_j)$

$$\mathrm{cd}_j = \mathrm{tcd}(z'_{j+1}) - \mathrm{tcd}(z'_j)$$

- Grid is optimized so mean values are located at output levels
- Errors are amplified through differencing integrated amount
  - (differentiating interpolation function
- Errors are more sensitive to functional form of interpolating functions
- Mixing ratios at endpoints P_1, P_n must be extrapolated from mean values

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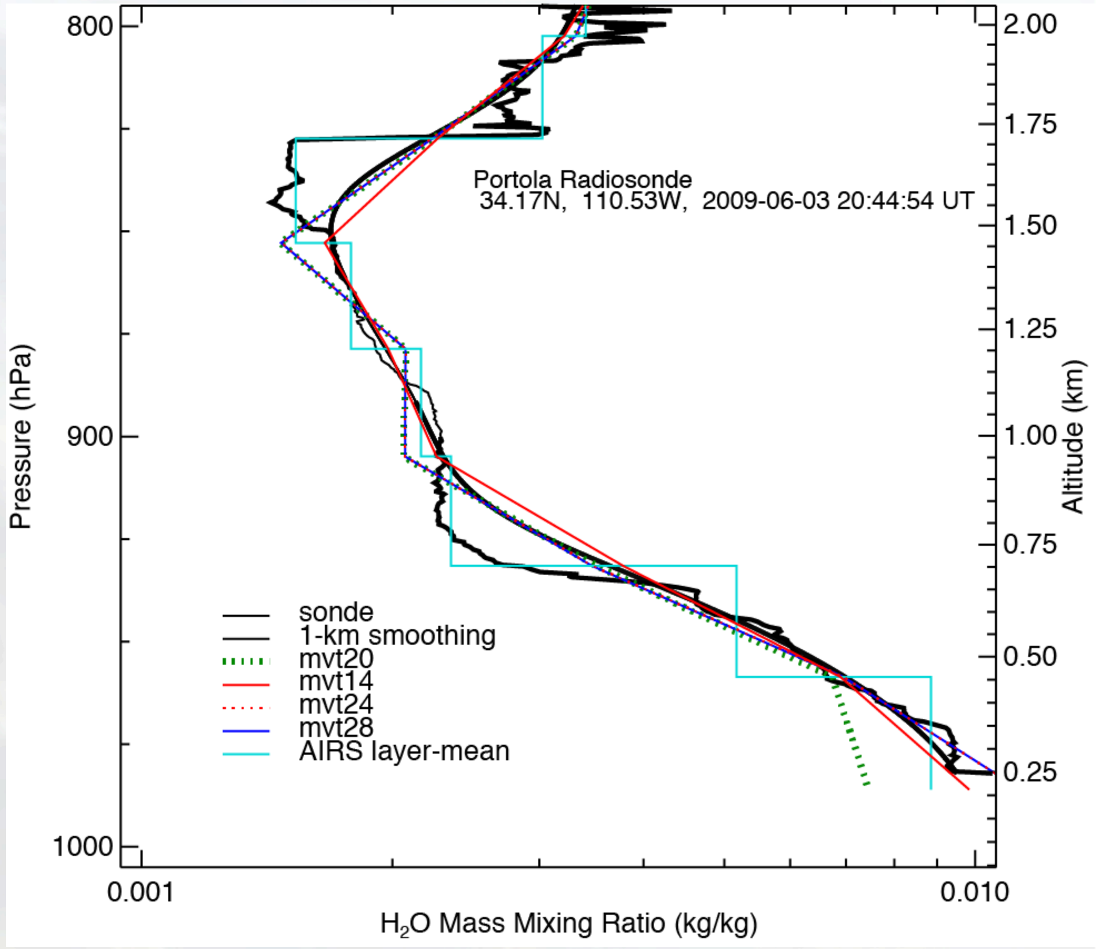
Extended MVT Approach Example

- Layer boundaries at arithmetic means of output levels
  - Linear interpolation (mvt14)
  - Cubic-spline interpolation (mvt24)
- Layer boundaries at logarithmic means of output levels
  - Cubic-spline interpolation (mvt28)

- Error Sources
  - Assumed position
  - Functional form of interpolating function
- Better near-surface mixing ratio

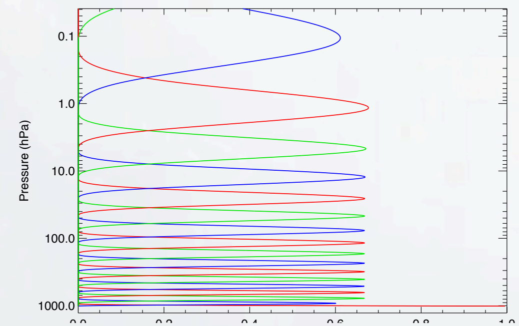


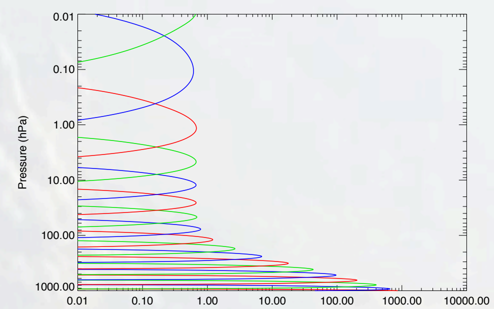
Function Fitting Method

- Represent the profile as a linear combination of functions

$$X(\zeta) = \sum_{i=0}^{n} a_i b_i(\zeta) \qquad \zeta = H_0 \, ln\left(\frac{P_{\text{Surf}}}{P}\right)$$

- *Functions* b_i can be
  - 2^{nd} order basis splines (triangles)
  - 4^{th} order basis splines (cubic splines)
  - EOF of water vapor climatology
  - Scaled basis splines
- Trade-off number versus complexity
- Fitting can be interpolating or smoothing





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Functional Fitting Method Formalism

- Profile:

$$X(\zeta) = \sum_{i=0}^{n} a_i b_i(\zeta)$$

$$\zeta = H_0 \; ln\left(\frac{P_{\mathrm{Surf}}}{P}\right)$$

- Layer amount

$$\begin{aligned} cd_i{}' &= \int_{P_i}^{P_{i-1}} X(P) \frac{dP}{g(z,\theta,\lambda)} \\ &= \sum_{j=0}^{M} a_j \int_{P_i}^{P_{i-1}} b_j(\zeta) \frac{dP}{g(z,\theta,\lambda)} \\ &= B_{ij} a_j = \mathbf{Ba} \end{aligned}$$

$$z(P) = \int_{P_{\mathrm{Surf}}}^{P} \frac{RT}{g(z,\theta,\lambda)} d\ln P$$

Least Squares Fitting

- Minimize

$$\chi^2 = (\mathbf{cd} - \mathbf{cd}')^{\mathbf{T}} \mathbf{C}^{-1} (\mathbf{cd} - \mathbf{cd}')$$

- Subject to constraint on total mass: $\sum_{i=1}^{M} cd_i = \sum_{i=1}^{M} cd'_i$
  - No significant change and places overly tight constraint on near surface layers which are poorly measured by IR/MW sounding
- Also introduce Bayesian constraints
  - Climatological profile with huge uncertainty
  - Climatological smoothness constraint

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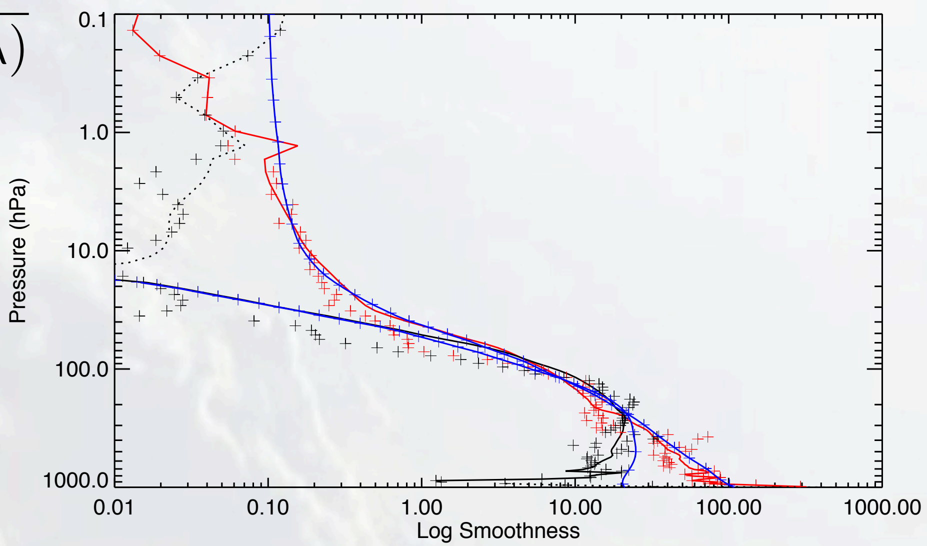
Bayesian Smoothness Constraint

$$\chi^2 = (\mathbf{cd} - \mathbf{cd}')^{\mathbf{T}} \mathbf{C}^{-\mathbf{1}} (\mathbf{cd} - \mathbf{cd}') + (\mathbf{s_a} - \mathbf{s}')^{\mathbf{T}} \mathbf{S}^{-\mathbf{1}} (\mathbf{s_a} - \mathbf{s}')$$

$$\begin{aligned} s_i{}' &= \int_{P_i}^{P_{i-1}} \partial_{\zeta\zeta} X(P) \frac{dP}{g(z,\theta,\lambda)} \\ &= \sum_{j=0}^{M} a_j \int_{P_i}^{P_{i-1}} \partial_{\zeta\zeta} b_j(\zeta) \frac{dP}{g(z,\theta,\lambda)} \\ &= S_{ij} a_j = \mathbf{Sa} \end{aligned}$$



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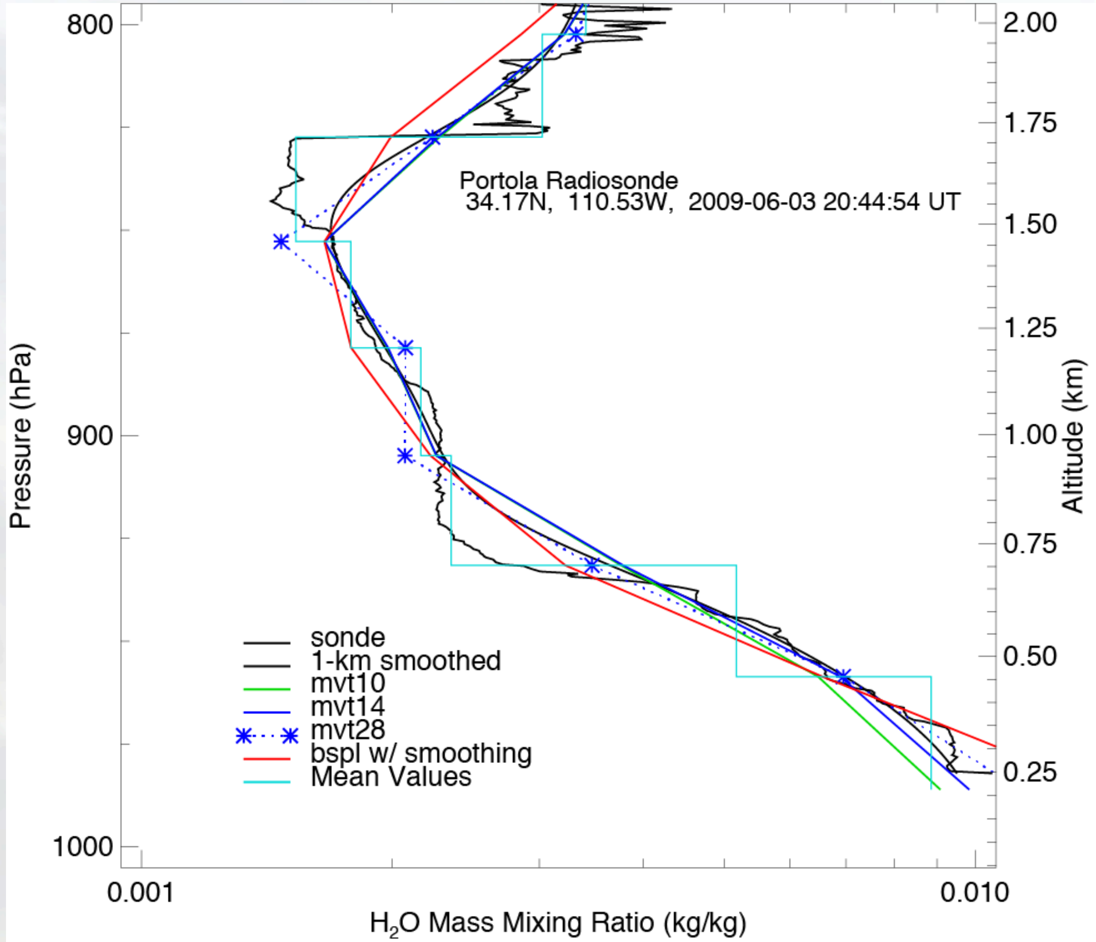
Spline Fitting Approach, Example

- Mean values with layers
  - Linear interpolation (mvt10)
- Layer boundaries at arithmetic means of output levels
  - Linear interpolation (mvt14)
- Layer boundaries at logarithmic means of output levels
  - Cubic-spline interpolation (mvt28)
- Cubic b-splines
  - Knots at every 3rd AIRS level
  - Climatological smoothing



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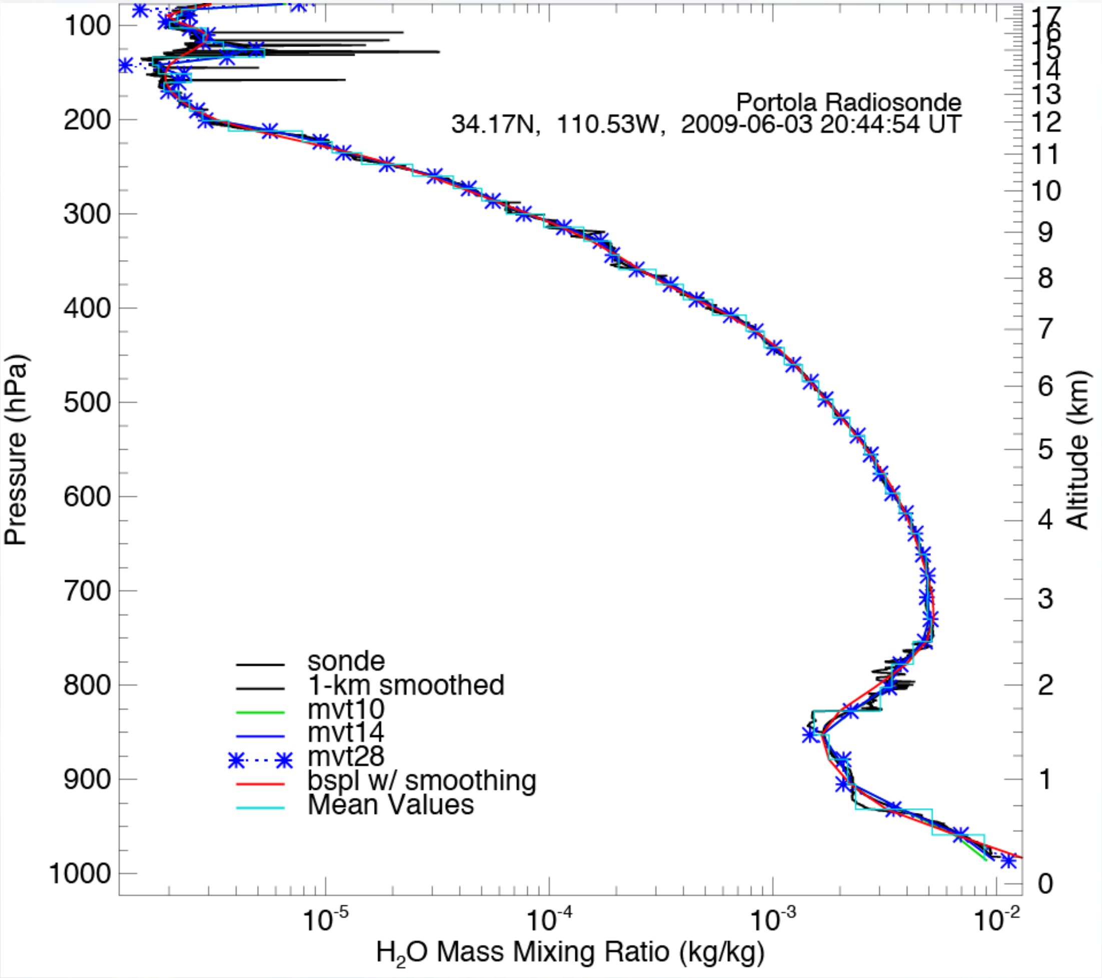
Summary – The Troposphere

- Same curves as last figure
- Over the full troposphere, all methods look similar, except
  - Cublic spline extended mvt near tropopause
  - Smoothing dampens oscillation near tropopause



Analysis over TIGR Ensemble of Radiosondes

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Data Set for analysis and testing

- ***Description of TIGR :***
  - ***TIGR dataset includes 2311 atmospheres classified in 5 airmass types :***
    - ***Tropical atmospheres 1 to 872***
    - ***Mid-latitude 1: 873 to 1260***
    - ***Mid-latitude 2: 1261 to 1614***
    - ***Polar 1: 1615 to 1718***
    - ***Polar 2: 1719 to 2311***
- ***Methodology***
  - ***Interpolate water vapor mmr profile to AIRS support levels (X_{truth})***
  - ***Convert water vapor mmr profile to layer amount (cd) on AIRS support levels***
  - ***Process layer amount profile through layers to levels routine cd -> X***
  - ***Compare X <--> X_{truth}***

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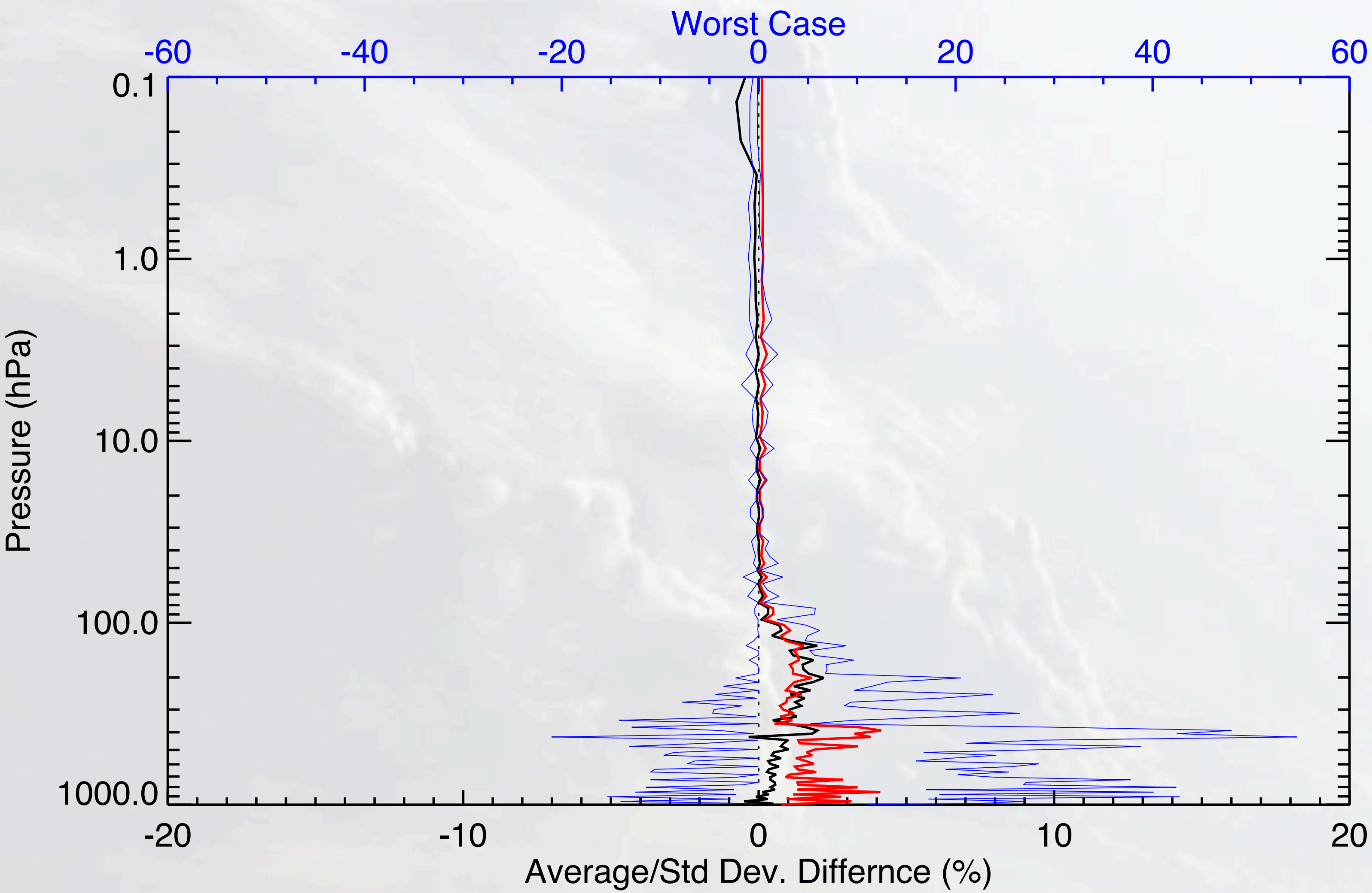
Mean Value Method Results – Profile Statistics



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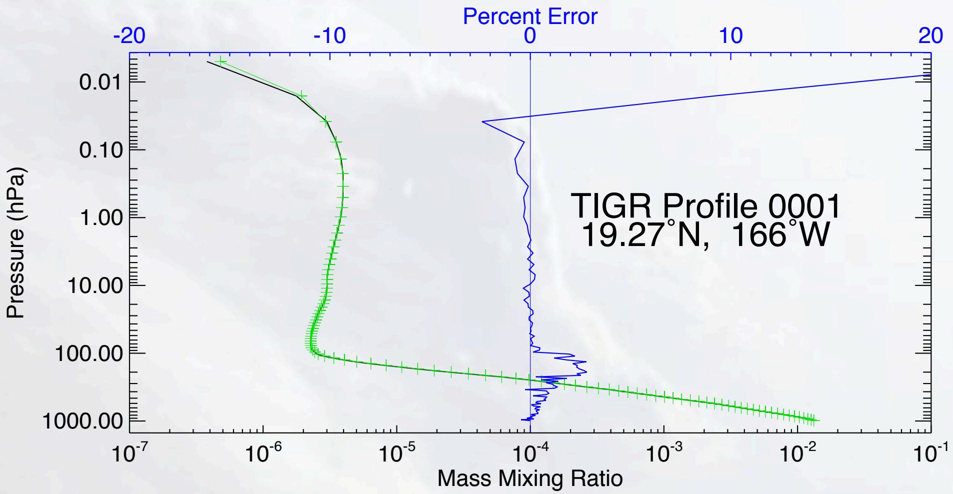
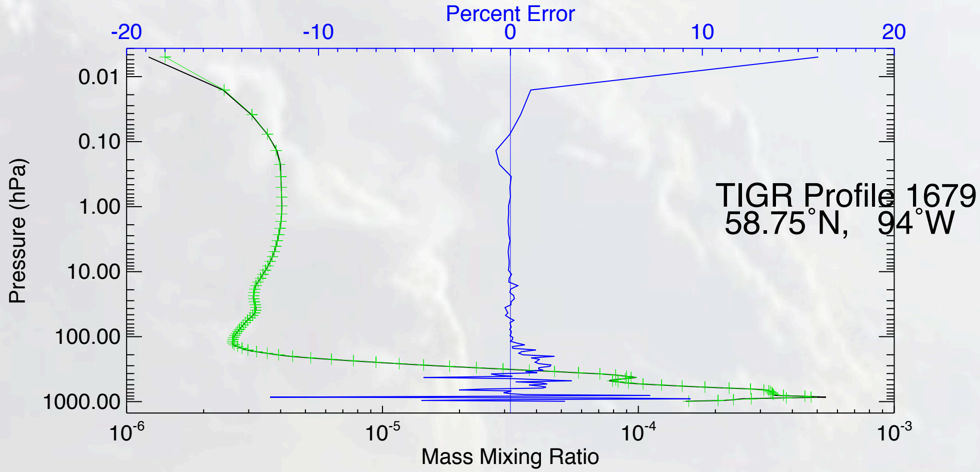
Mean Value Method Results – Sample Profiles





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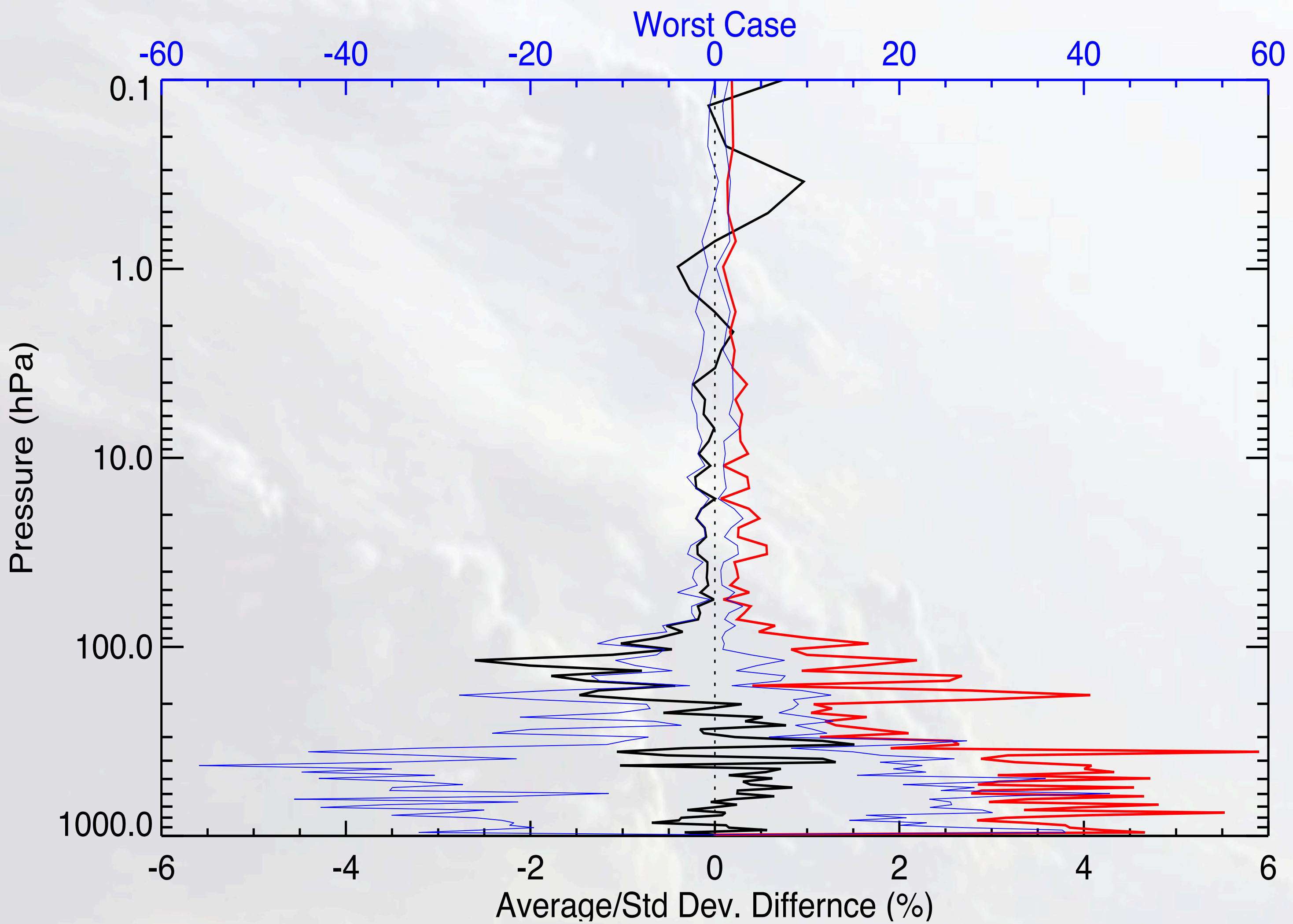
Functional Fitting Profile Statistics



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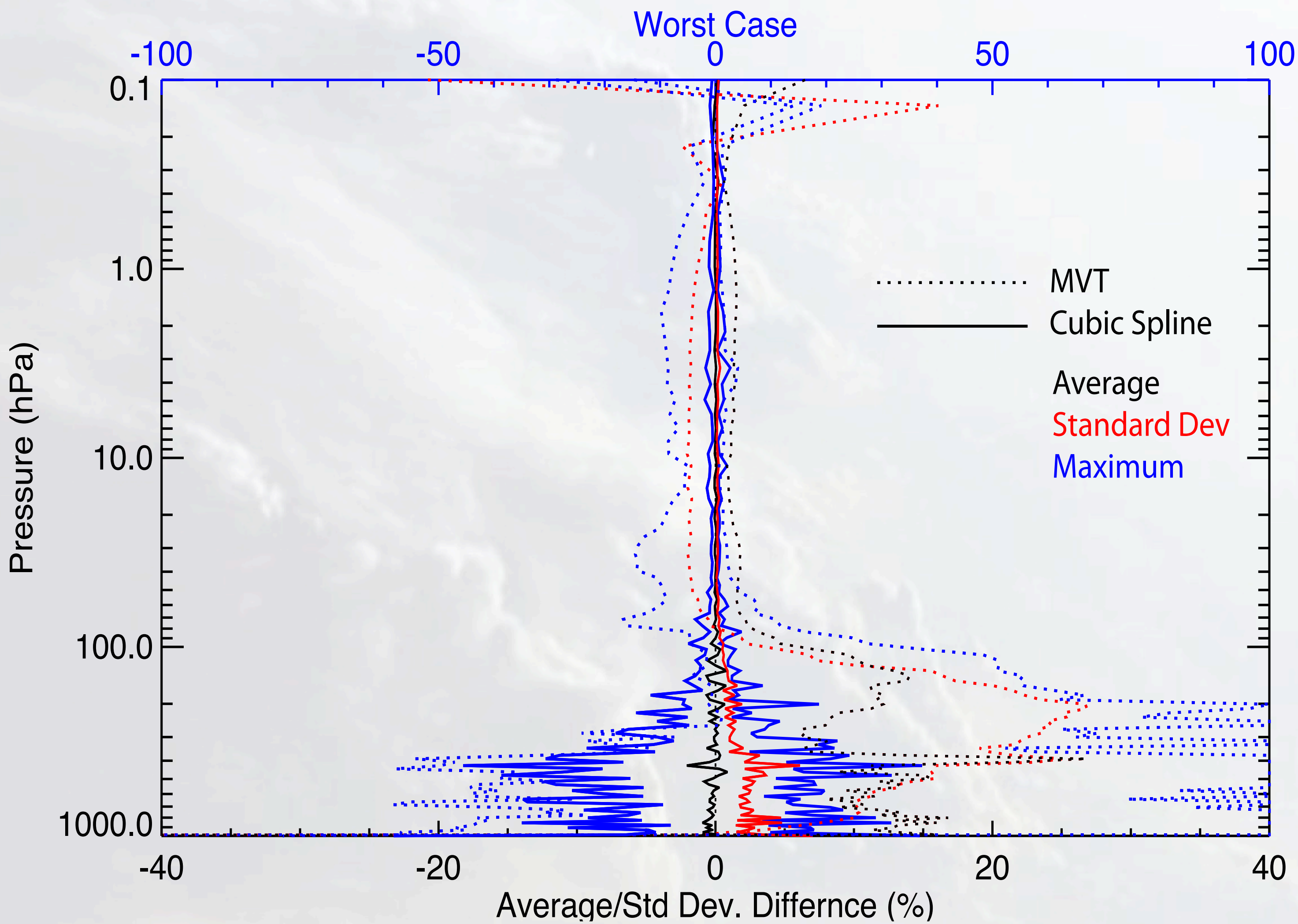
Functional Fitting Profile Compared with MVT



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Summary

- All methods have tunable parameters, constrained by information content, vertical resolution, noise correlation
- This analysis only considers the vertical resolution constrained by layering
- 10-20% error in water vapor from conversion to vertical profiles.