Scene in-homogeneities effects on interferometer-based radiance measurements and their impact on retrievals

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Introduction

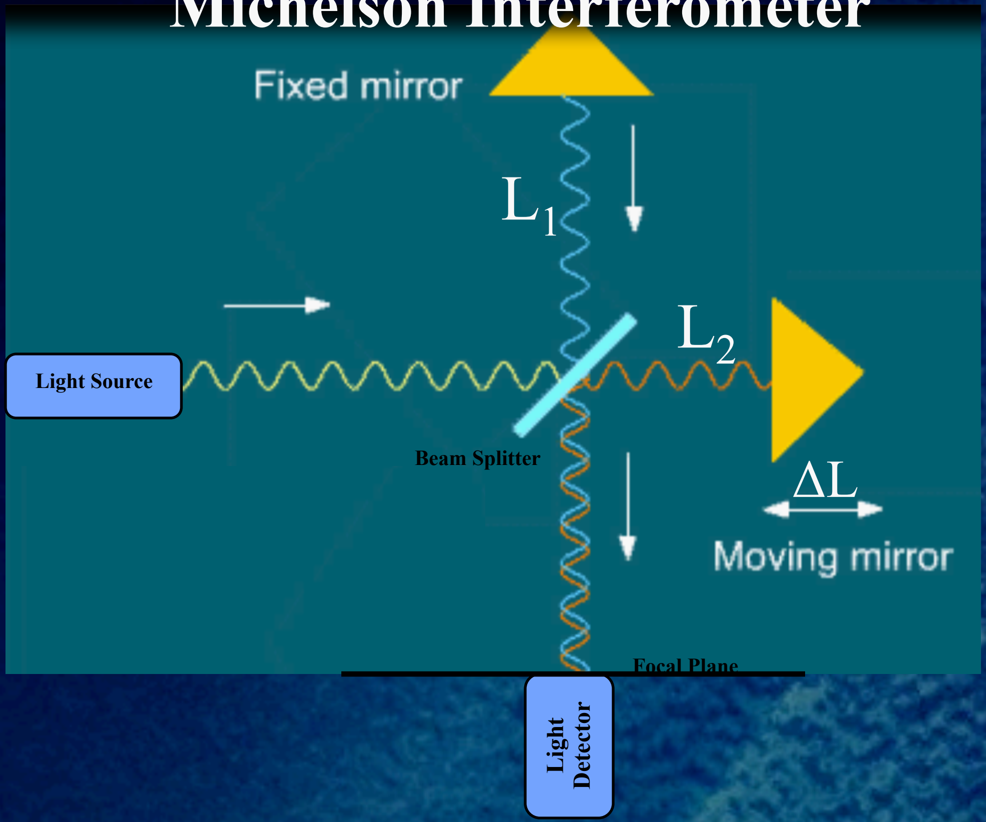
- **The precise knowledge of the Instrument Line Shape (ILS) of an interferometer is critical for any application of the radiance measurement: any error in the knowledge of the ILS will introduce a radiance error *δR* in the radiance spectrum.**

- **Scene in-homogeneities (clouds, surface in-homogeneities over the field of view) are responsible for an overall distortion of the theoretical FOV ILS, which is mainly a peak frequency shift effect, *δv*, hence the definition of "ILS shift".**

- ***1) What is the magnitude of the radiance error introduced by the ILS distortion?***
- ***2) What is the impact on the retrieval accuracy?***

Basic Concept of Interferometry

Michelson Interferometer



The detector measures the variation of intensity as the mirror is displaced:

$$G(x) = g(\nu)[1 + \cos(2\pi\nu \cdot x)]$$

g(ν) = Input radiant power at frequency ν
x = Path Difference = 2(L1-L2)
Constructive Interference: x = nλ
Destructive Interference: x = (2n+1) λ

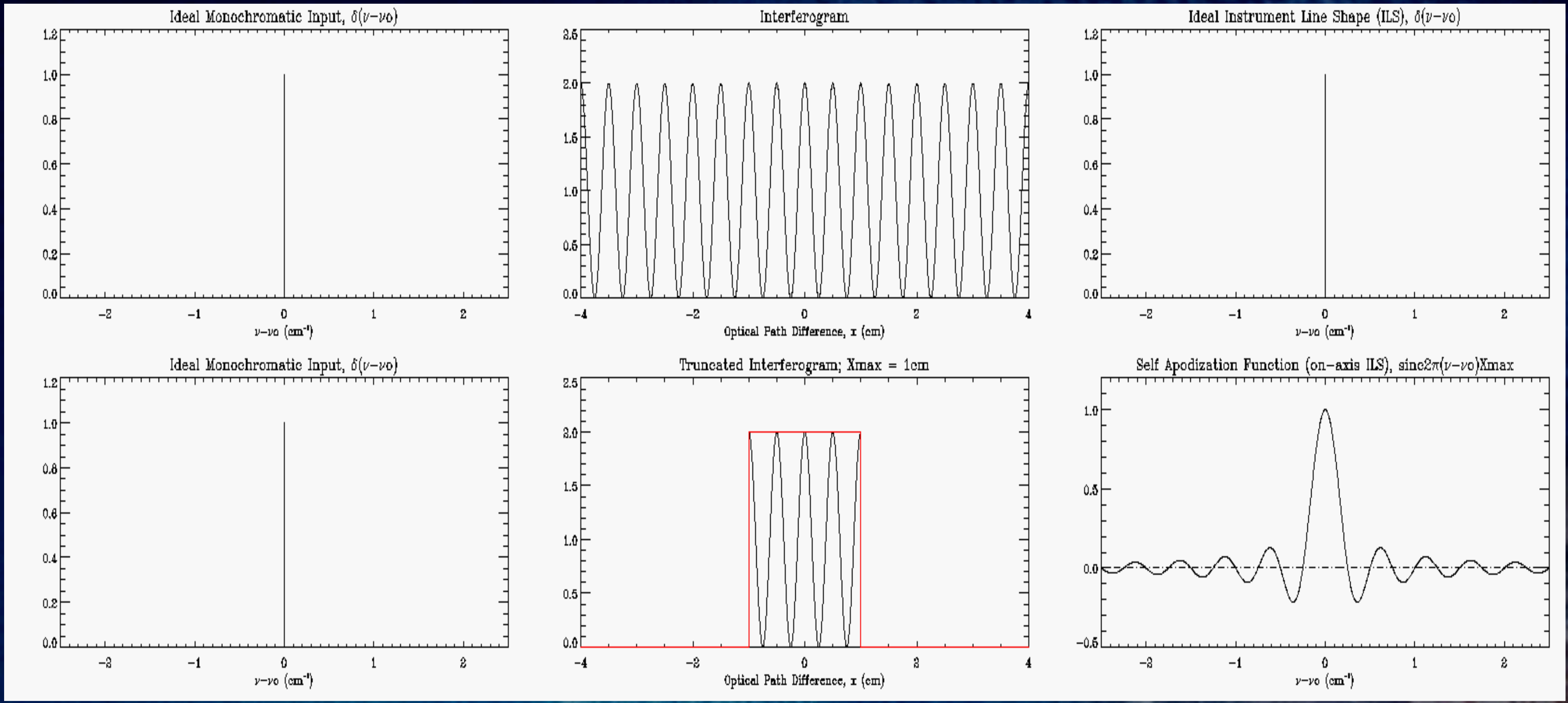
Polychromatic source:

$$G(x) = \int_0^\infty g(\nu)[1 + \cos(2\pi\nu \cdot x)]d\nu$$

Interferogram = oscillating part of G(x)

$$I(x) = \int_0^\infty g(\nu)\cos(2\pi\nu \cdot x)d\nu$$

Truncation of the Interferogram & Resulting Instrument Line Shape



The Instrument Line Shape resulting from the box-car truncation is a sinc function with pronounced side lobe effects.

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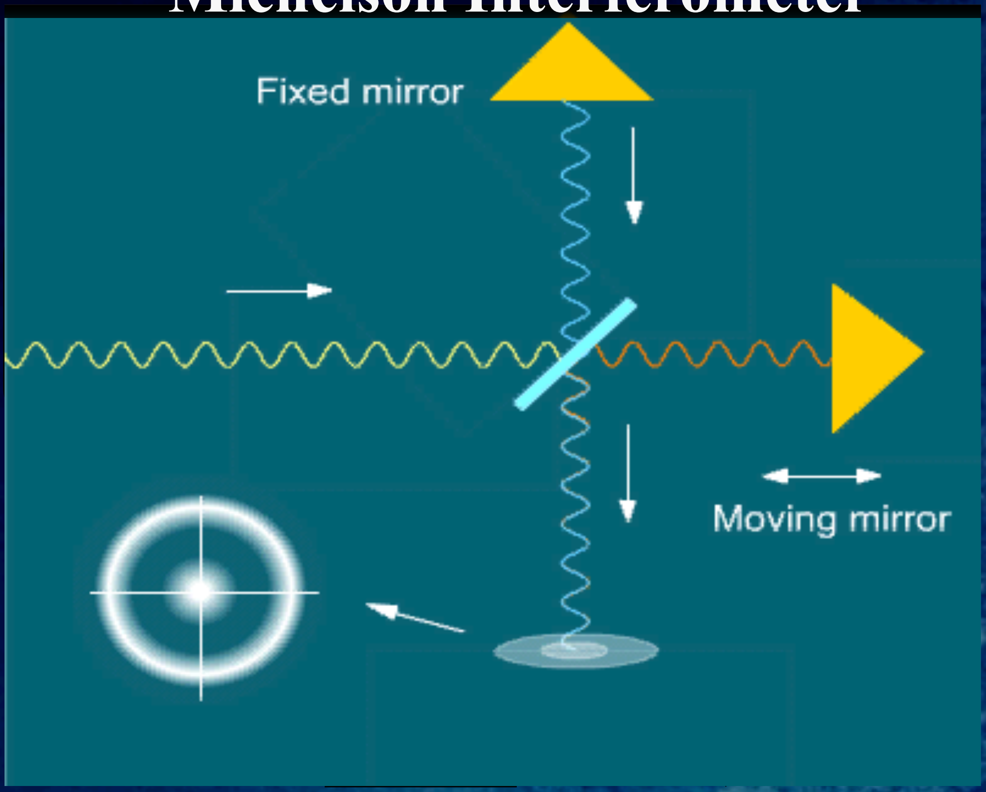


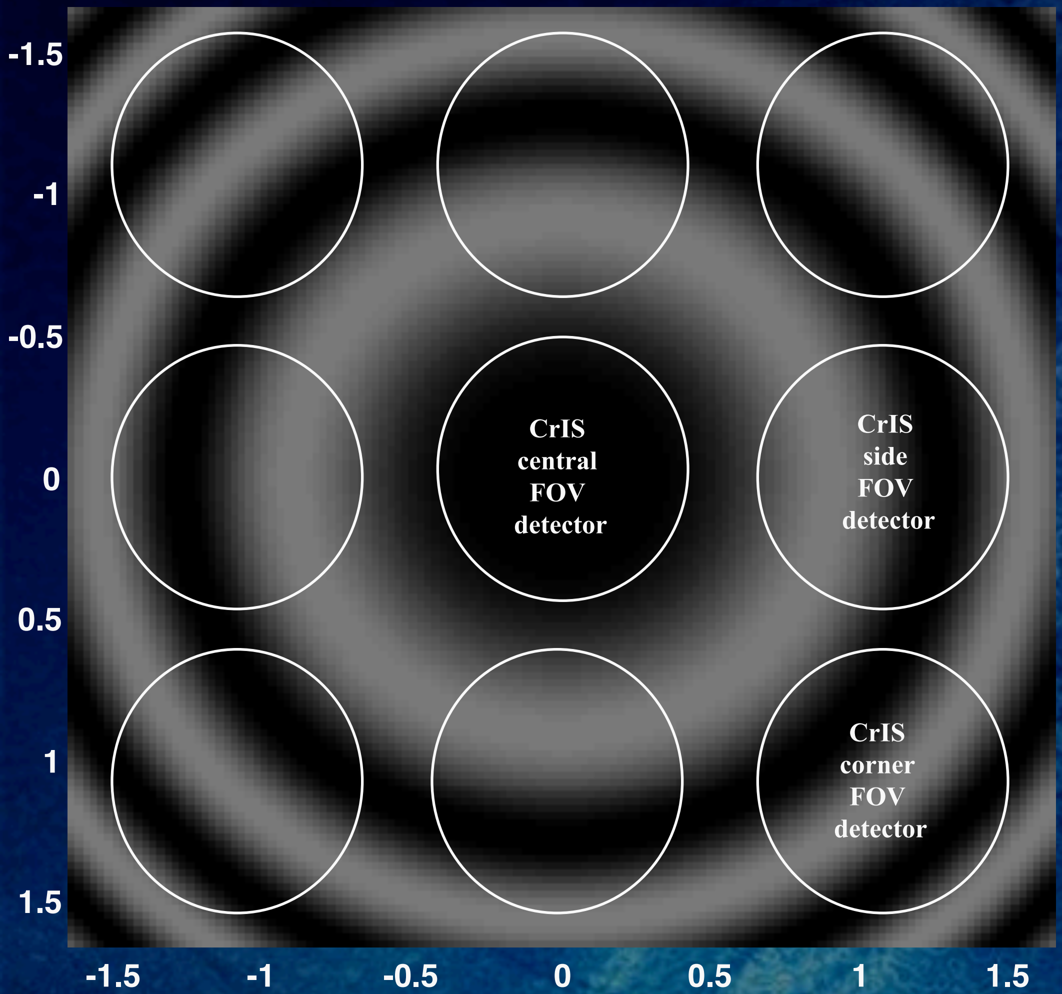
A natural source of light has off-axis propagating beams which will intercept the focal plane at different angles, α.

Off-axis optical path difference:
$X_{off\text{-}axis} = X_{on\text{-}axis} \cos\alpha$

The off-axis measurement on the screen is a α-dependent interference pattern consisting of a bright center and alternating dark and bright fringes given by:

$$G(x) = g(\nu)[1 + \cos(2\pi\nu \cdot x \cos\alpha)]$$

Self Apodization Effect & CrIS FOV Geometry

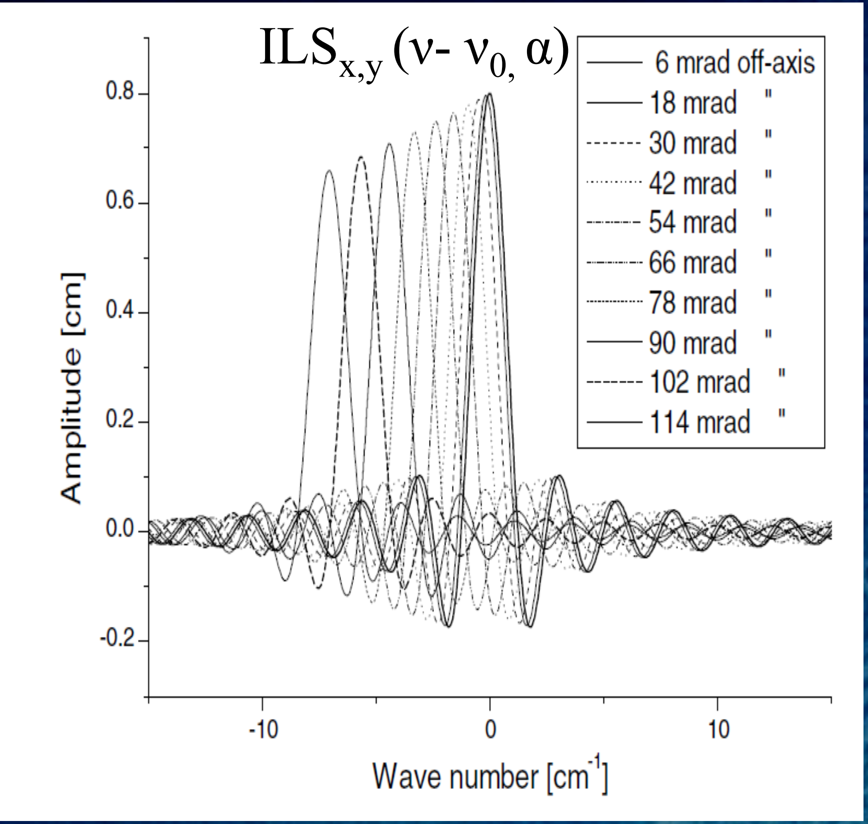


What the detector measures is the integration over the solid angle subtended by the detector at the exit pupil:

$$G(x) = \int_{0}^{\Omega_{max}} G(x,\Omega)\,d\Omega$$

If the detector FOV falls beyond the central bright spot, it will integrate over bright and dark fringes, hence a signal loss corresponding to a reduction in signal to noise ratio ("Self Apodization Effect")

Off-Axis ILS



The off-axis ILS is shifted, asymmetric and attenuated. The frequency shift of the peak is the dominating effect and is related to the angular offset of the light beam as:

$$\frac{\delta \upsilon}{\upsilon} \sim \alpha \delta \alpha$$

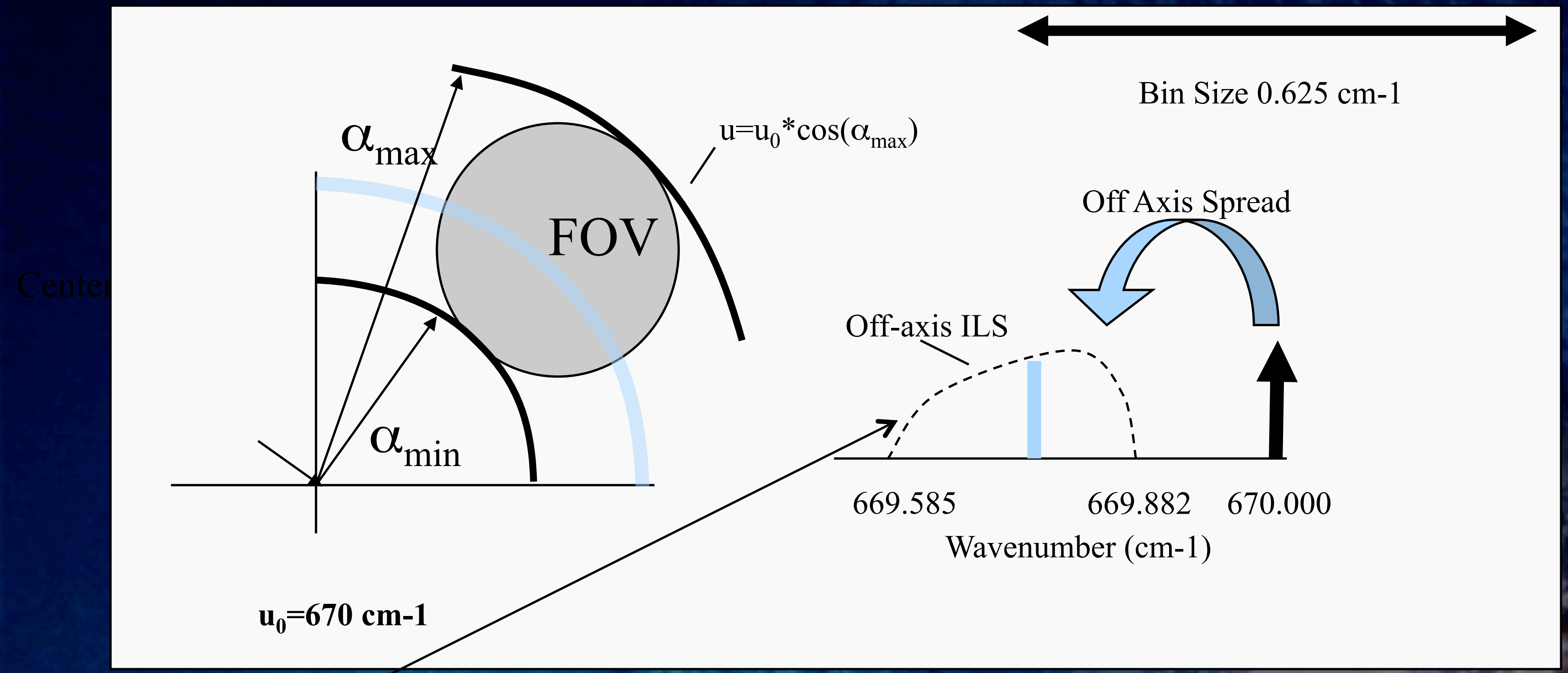
Homogeneous source (and monochromatic):

$$\mathrm{ILS}^{\alpha}{}_{x,y}\left(\nu - \nu_0\right) = \mathrm{ILS}^{\alpha}{}_{x',y'}\left(\nu - \nu_0\right)$$

We can express the resulting output spectrum at each frequency as:

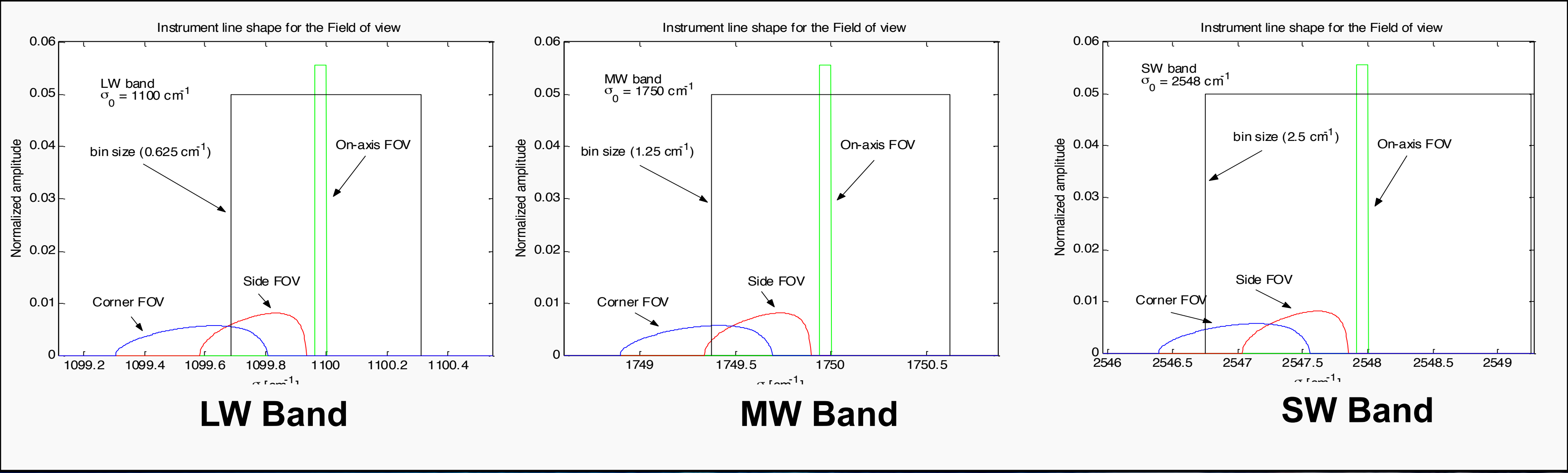
$$g_{meas}(\nu) = \sum_{FOV} ILS_{x,y}(\nu - \nu_0) \otimes g(\nu_0)$$

Smear & Shift Effect in an OFF-Axis FOV ILS



$$ILS_{FOV}(v - v_0) = \sum_{FOV} ILS_{x,y}(v - v_0)$$

Smear & Shift Effect of Each FOV ILS in the 3 Bands (one example for each band)



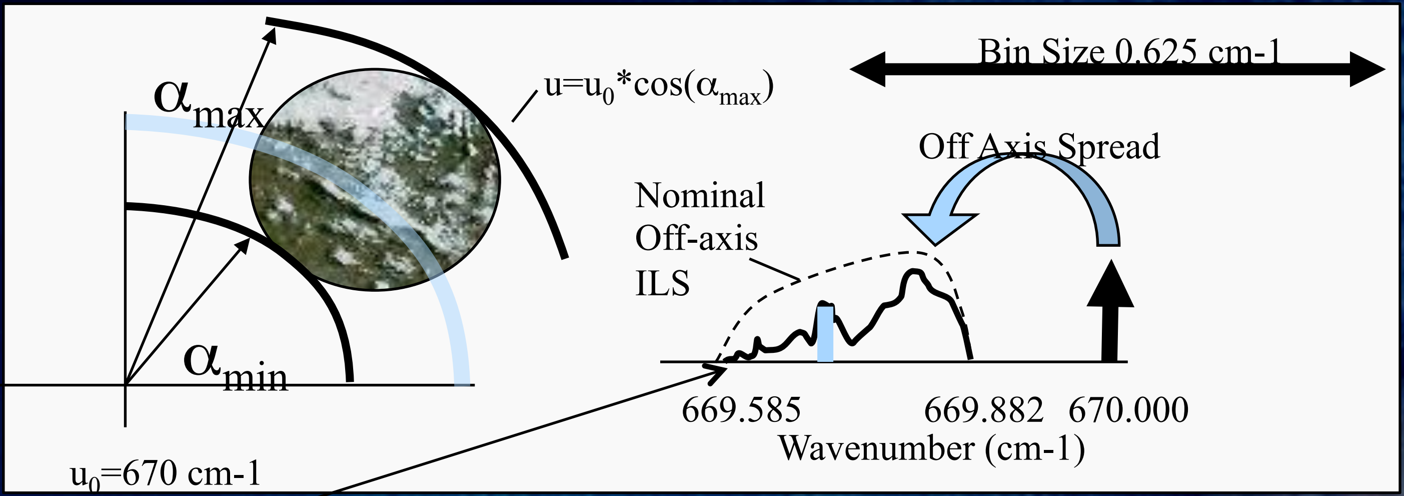
Picture courtesy of D. Mooney

- Shifts of the ILS by half a bin width are typical for corner FOVs

FOV ILS Distortion in Presence of Scene Inhomogeneities

In-homogeneous scene:

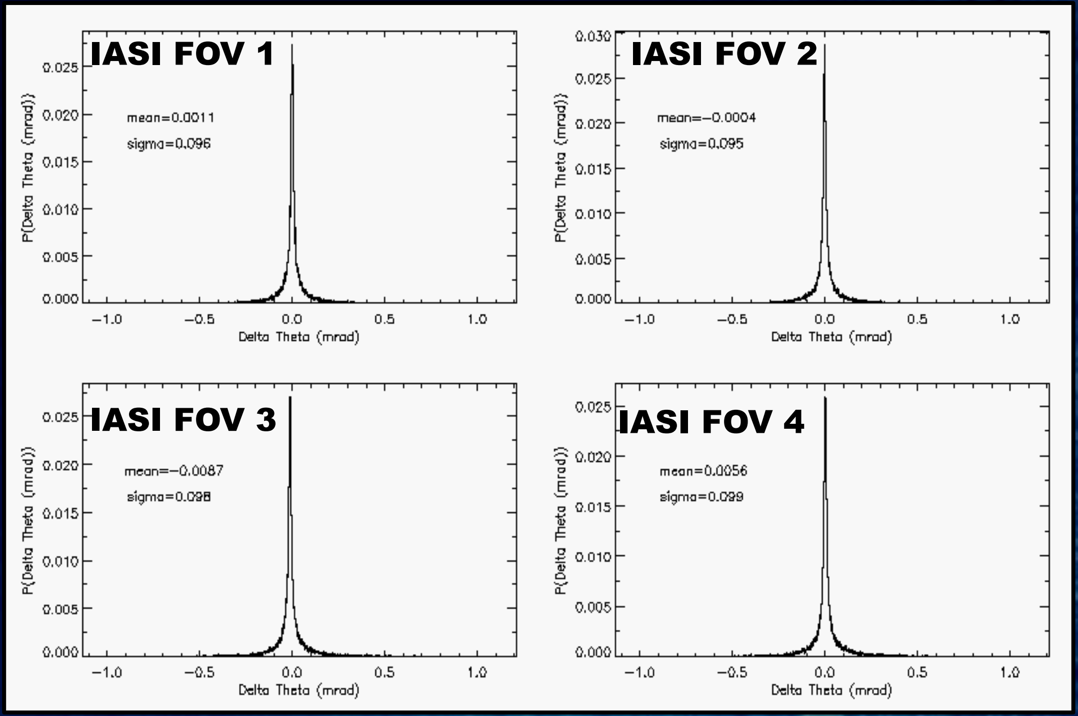
$$ILS^{\alpha}{}_{x,y}\left(\nu-\nu_0\right) \neq ILS^{\alpha}{}_{x',y'}\left(\nu-\nu_0\right)$$



$$ILS_{FOV}\left(\nu-\nu_0\right) = \sum_{FOV} ILS_{x,y}\left(\nu-\nu_0\right)$$

- Scene in-homogeneities (clouds, surface variability, et.) are responsible for an *angular* shift of the radiometric center of the FOV (towards the location in the FOV where the warmer scenes are distributed) and an associated distortion of the nominal FOV ILS. This introduces an error in the nominal self apodization matrix which mainly consists in a spectral shift of the FOV ILS peak frequency.
- This error is propagated through the off-axis correction (inversion of the self apodization matrix) introducing a signal attribution error in the radiance spectrum.

ILS frequency shift computation in presence of non uniform scenes: lessons learned from IASI



The ILS distortion due to the presence of scence inhomogeneities is mainly a frequency shift effect,δν . Its relationship with the angular offset,δα, between the geometric and radiometric centers of the FOV is :

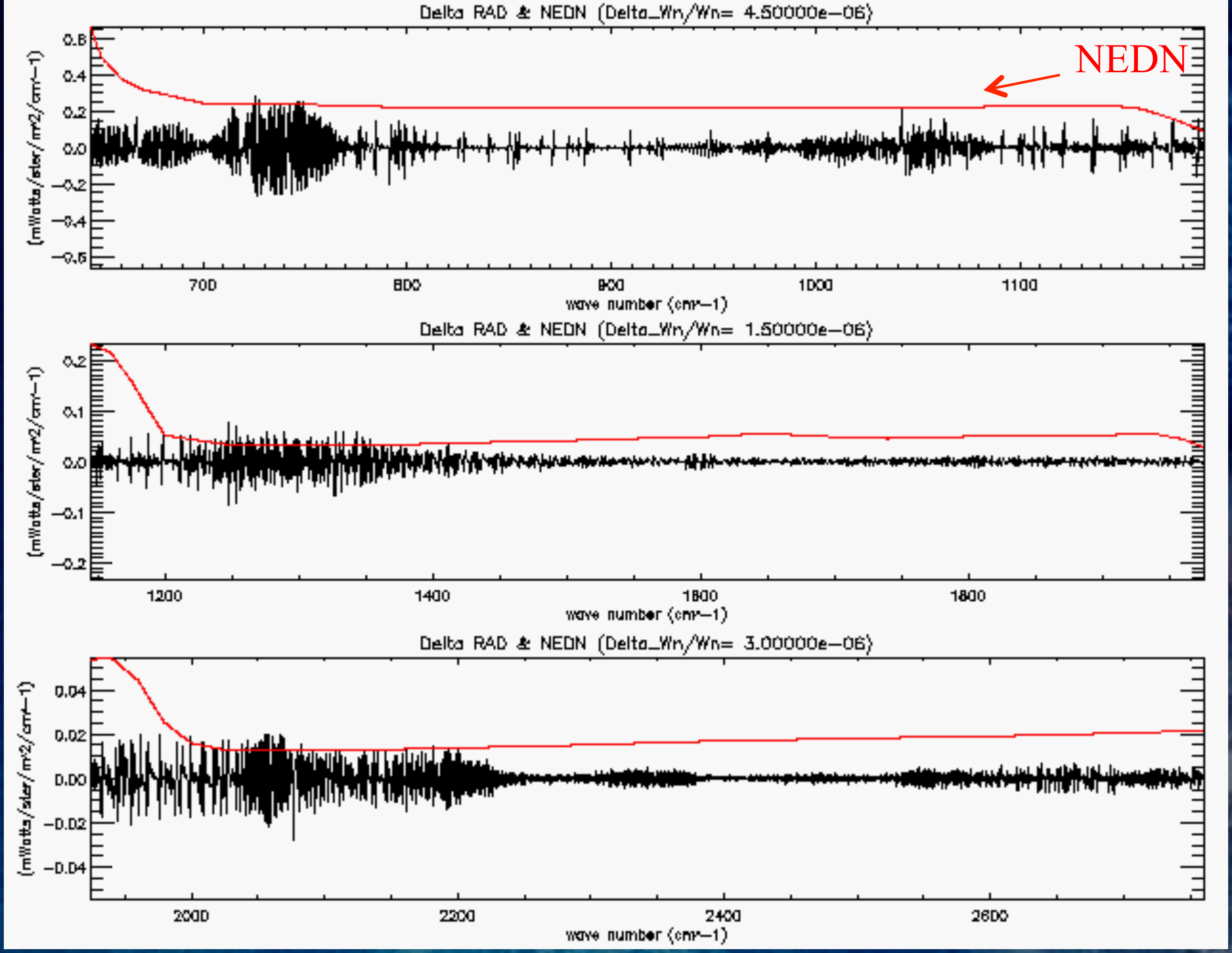
$$\frac{\delta\nu}{\nu} \sim \alpha_0 \delta\alpha$$

Lessons learned from IASI + IIS:

- Global δα distribution results:
  mean = 0.001mrad;
  1 sigma = 0.1 mrad;
- Spectral shift:
  δν/ν =1.5 ppm (for δα = 1 sigma)
- Radiance error lower than NEDN across the three bands, hence is negligible (next slide).

Ref: Gambacorta et al.; Proceedings of 2nd IASI International Meeting, Sevrier, 2010.

IASI radiance error induce by ILS shift



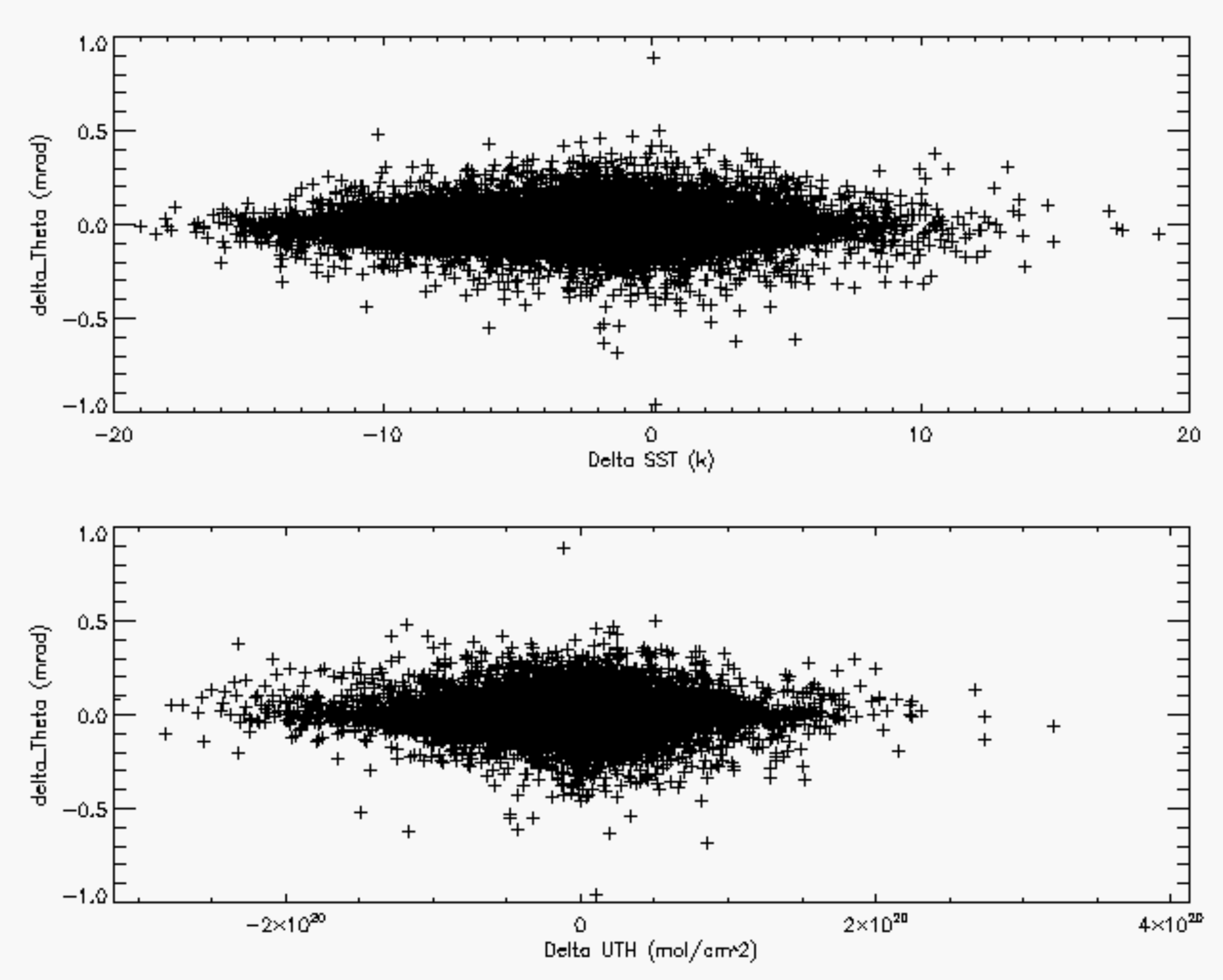
Band 1
3 sigma shift error

Band 2
1 sigma shift error

Band 3
2 sigma shift error

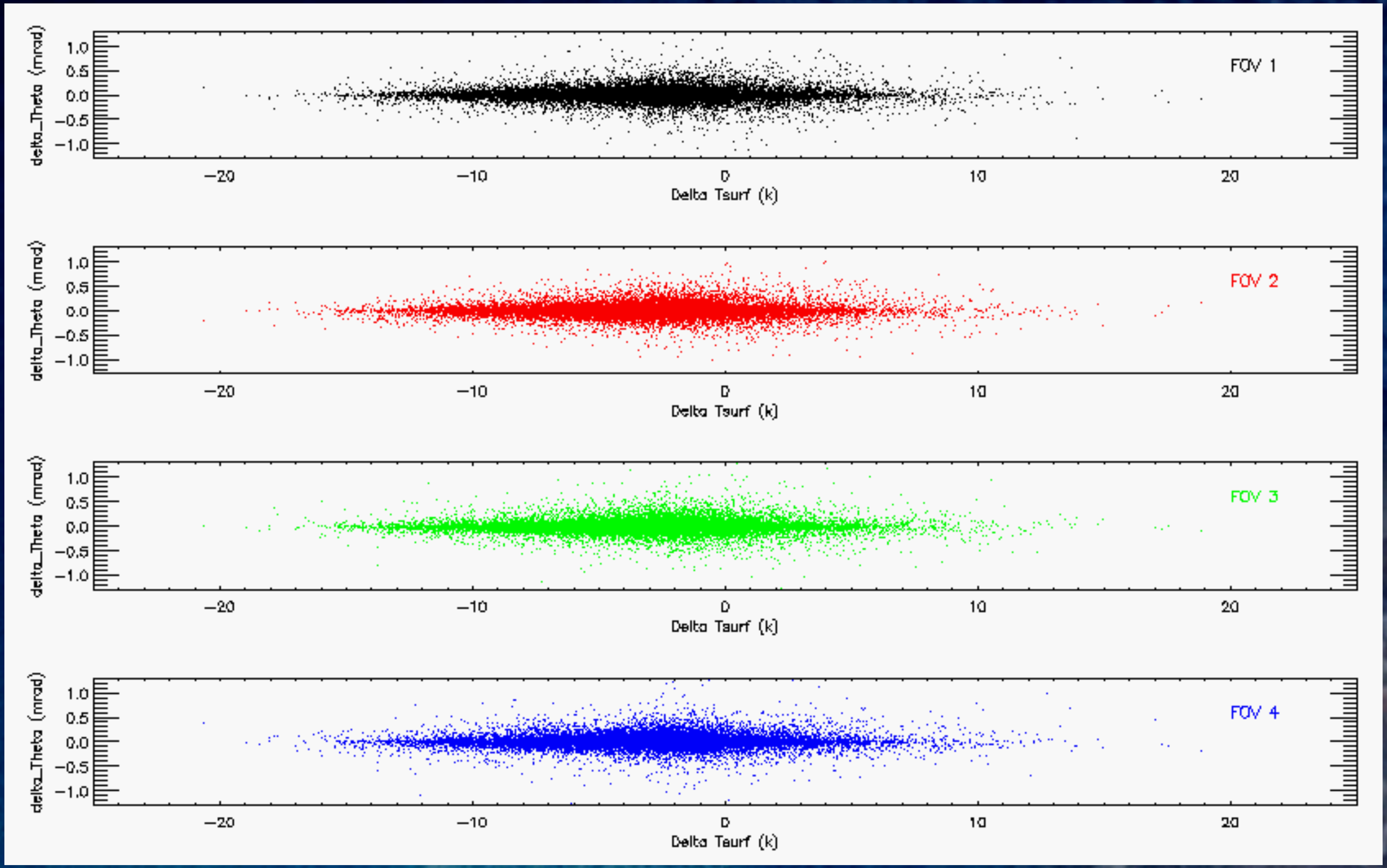
- **Radiance error lower than NEDN across the three bands, hence is negligible.**

4 FOV MEAN Centroid Shift vs SST & UTH bias (Oct 19 2007)



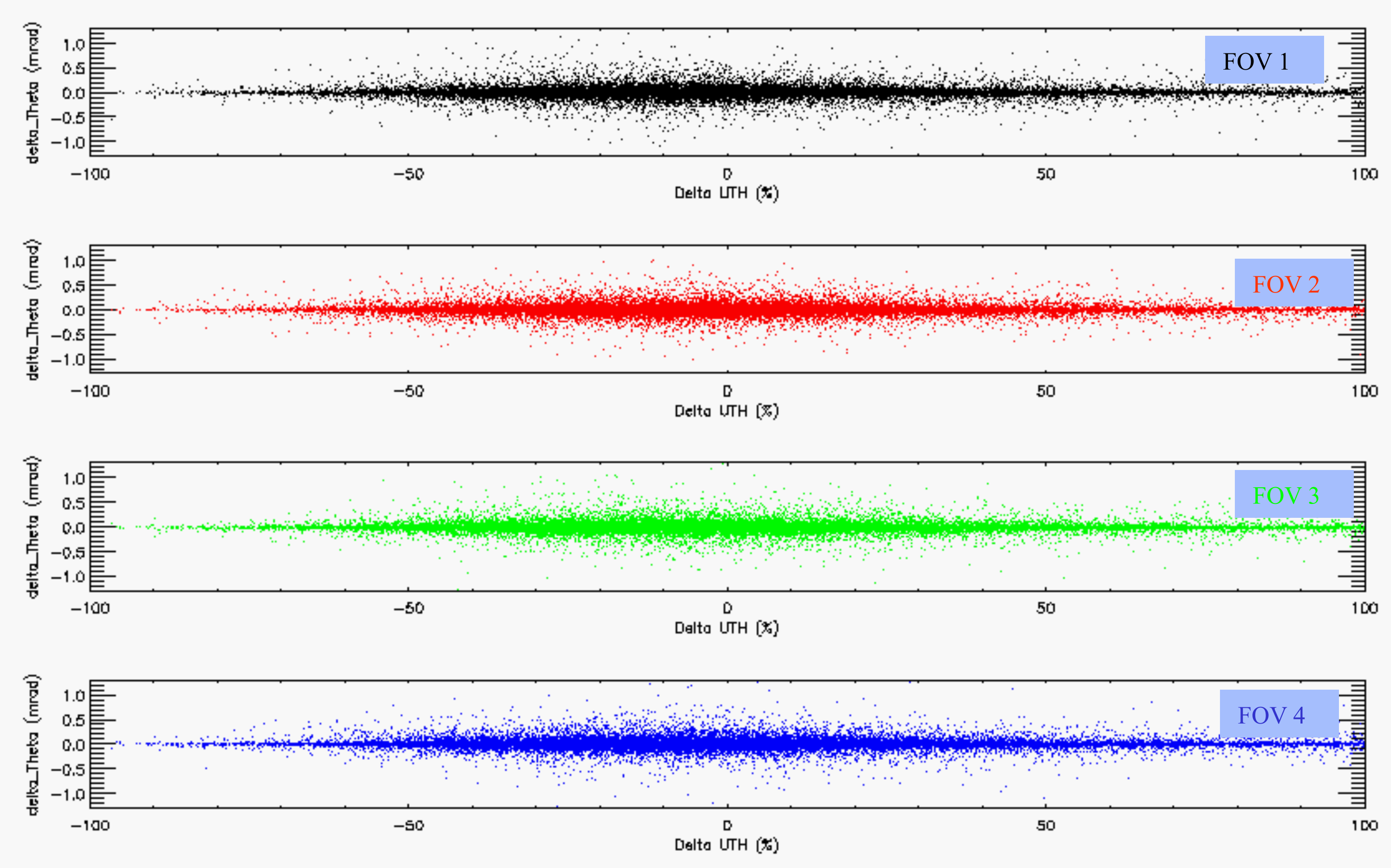
No significant correlation is seen to stand out between the averaged radiometric angular shift and the retrieved SST or UTH bias

Centroid Shift vs Tsurf bias (ret- ecmwf) (Oct 19 2007)



No significant correlation is seen to stand out between each FOV radiometric angular shift and the retrieved SST bias

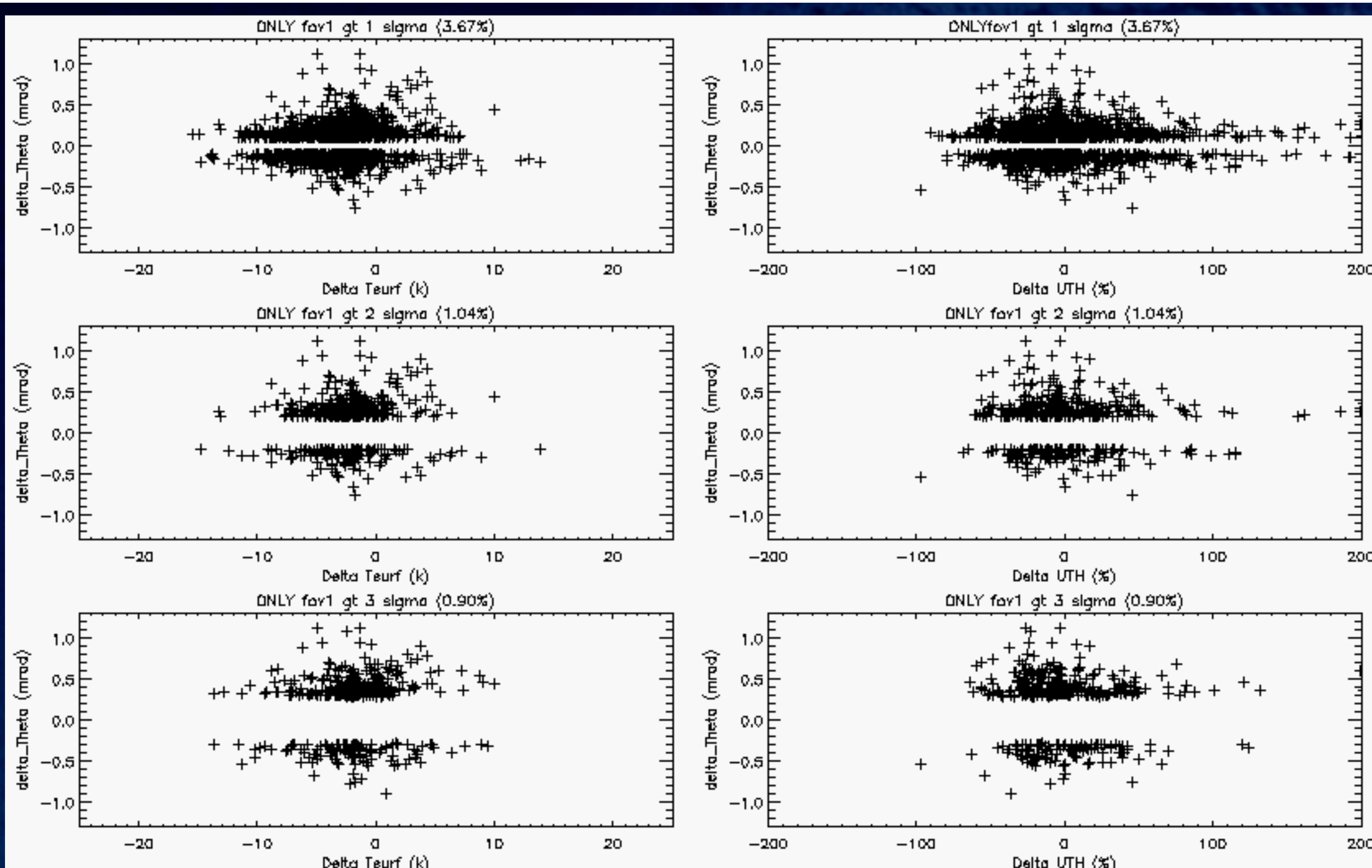
Centroid Shift vs UTH bias (ret- ecmwf) (Oct 19 2007)



No significant correlation is seen to stand out between each FOV radiometric angular shift and the retrieved UTH bias

Examples of cases that are likely to pass Radiance Cloud Clearing QAs (high cloud contrast)

1) ONLY FOV 1 has dα gt 1, all others lt 1 sigma
2) ONLY FOV 1 has dα gt 2, all others lt 1 sigma
3) ONLY FOV 1 has dα gt 3, all others lt 3 sigma



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Examples of cases that are likely to pass Radiance Cloud Clearing QAs (high cloud contrast)

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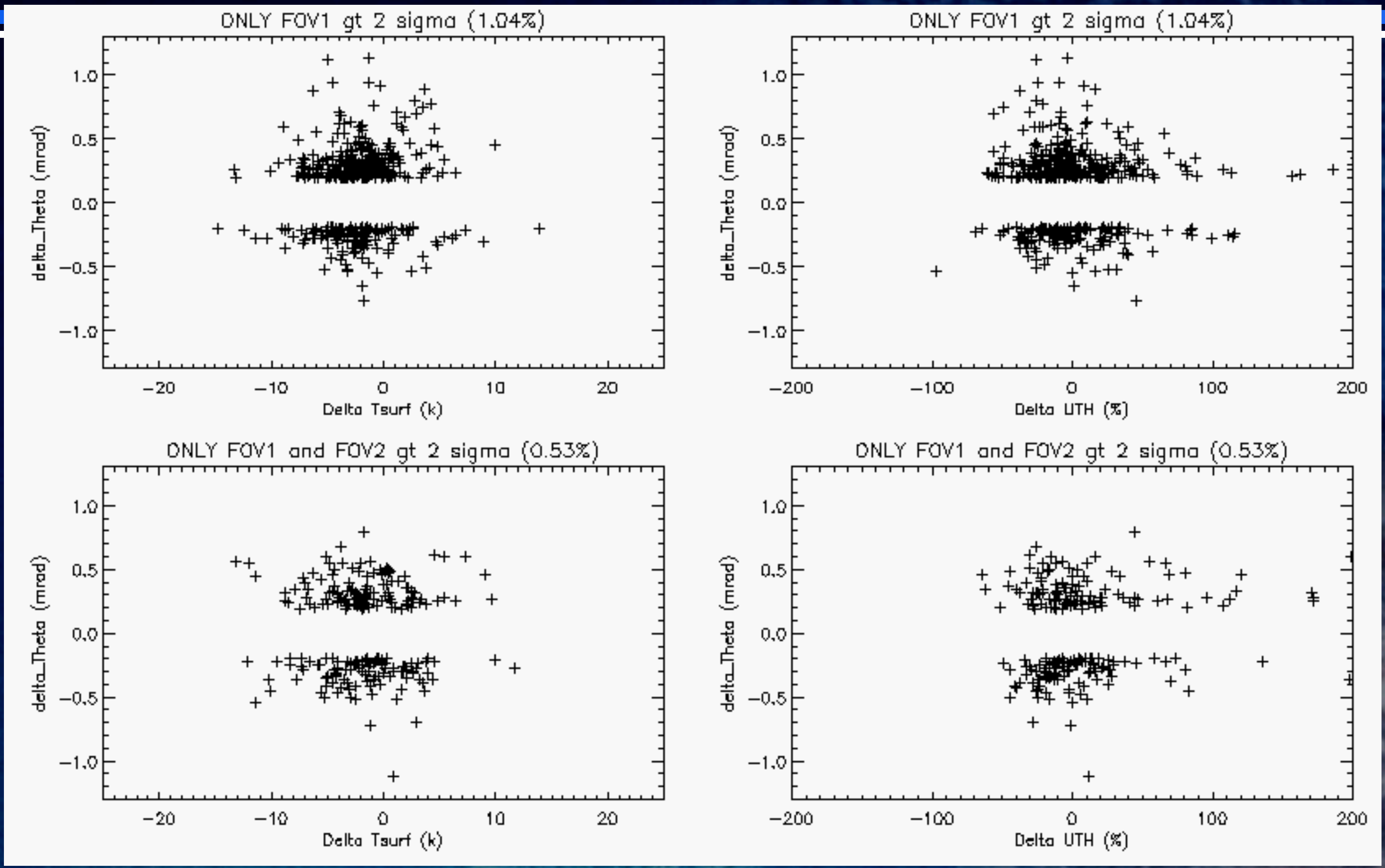

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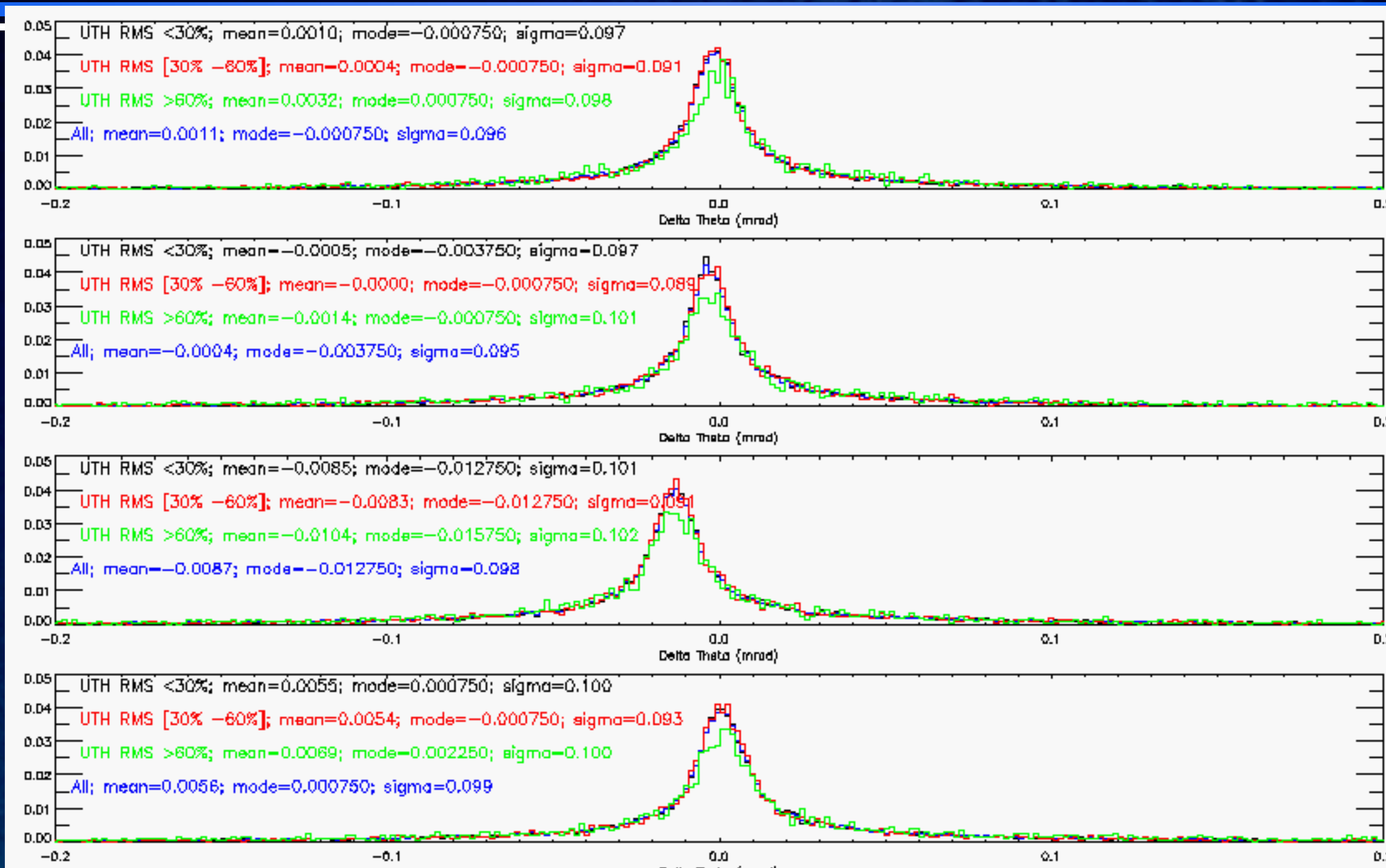
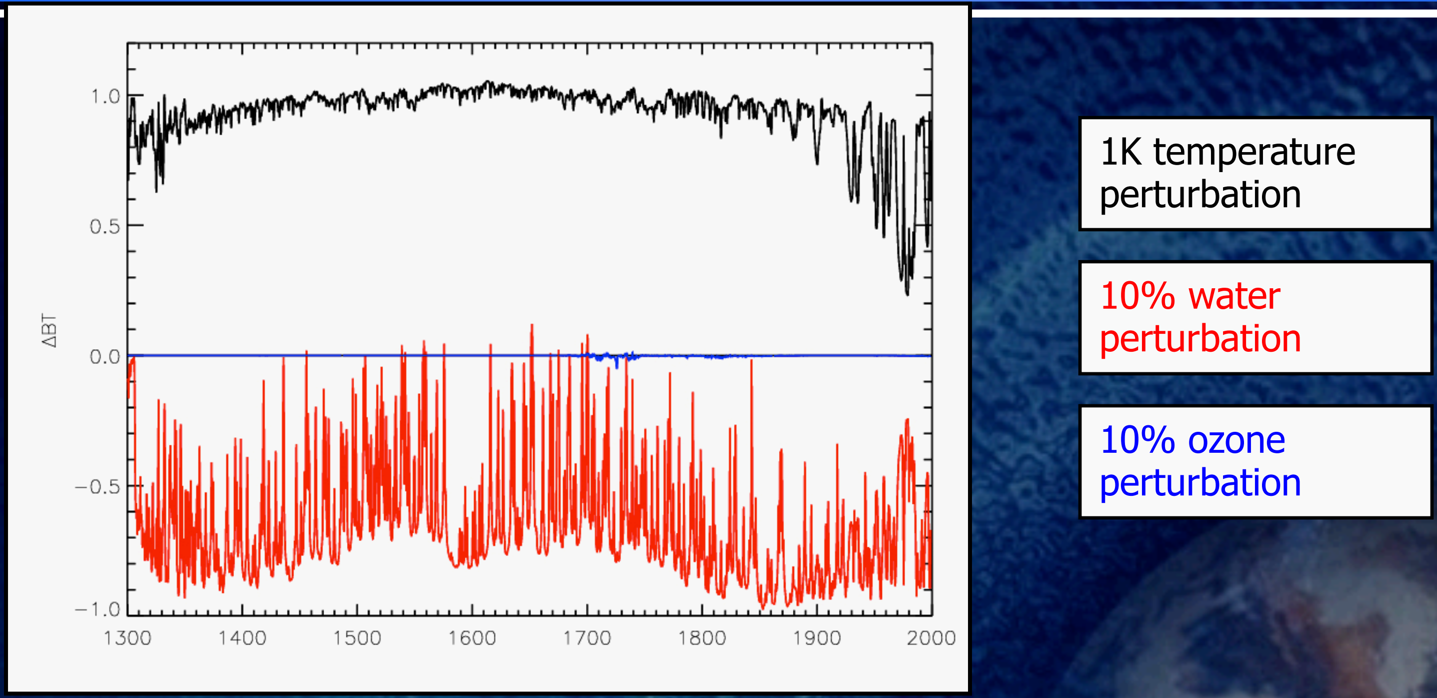
Centroid Distribution Conditioned by UTH statistics



No significant correlation is seen to stand out between the radiometric angular shift and the retrieved UTH rms

Comparison with the Sensitivity to Temperature and Water Vapor Perturbations in 6.7 μm Band



- The retrieval uncertainty appears to be dominated by other sources of error
- The main assumption of the cloud clearing algorithm is that besides clouds, everything in the FOR scene is homogeneous. This is a much broader assumption than the unperturbed ILS one; i.e. water vapor in the FOR can vary up to 10% and more. The radiance error introduced by this assumption can go up to 1K.

Radiance Error Assessment & Impact on the Retrieval Accuracy: Lessons Learned from IASI

- The analysis above indicates that the IASI radiance error induced by the ILS shift in presence of clouds is negligible:
  - Only 5% of the full day ensemble is seen to undergo an angular shift of ~1 sigma or higher.
  - The radiance error is by far smaller than the instrument noise for radiometric center offset values up to 3 sigma (band 1), 2 sigma (band 3) and 1 sigma (band2) of the overall offsets distributions.
  - In retrieval space, there does not appear to exist any correlation among angular offsets and retrieval biases of SST, UTH, CH4, etc (not shown) wrt ECMWF or climatology. This is possibly due to:
    - the presence of other factors dominating the uncertainty in the retrievals
    - no preferential distribution in angular offsets across the 4 FOVs (all 4 are centered around zero angular offset) such that the effect is likely to be averaged to zero during cloud clearing.
  - Angular offsets can still be monitored in order to build an ad hoc rejection flag (under study).

Lessons Learned from IASI and Considerations on the ILS Shift Effect on CrIS

•CrIS has lower instrument noise than IASI (the lower the max optical path, the lower the fringe effect, the higher the signal to noise), but a lower spectral resolution (the lower the max optical path, the lower the spectral resolution) which makes it less sensitive to the spectral shift.

• CrIS central FOV falls in within the central bright spot at all frequencies. Self apodization is more severe in IASI which makes it more sensitive to the ILS shift than CrIS.

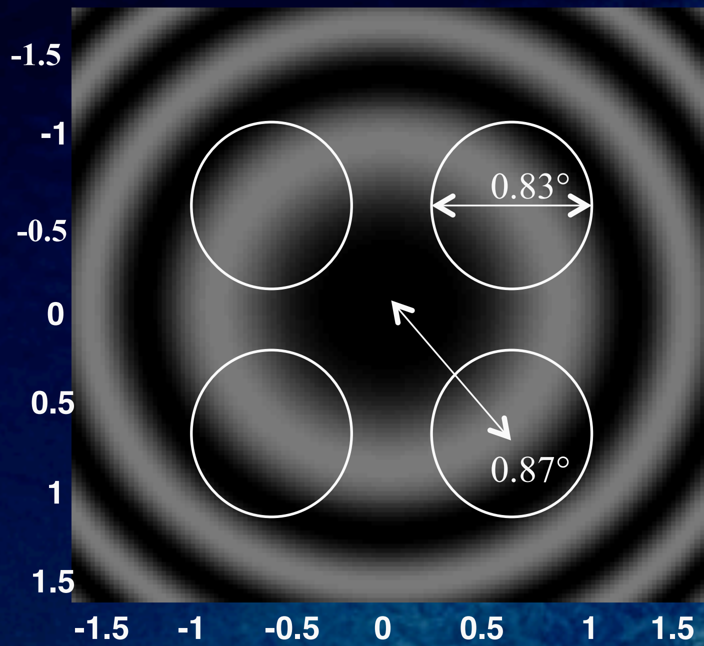
• IASI is a 9:am/9:30pm equatorial crossing orbit; CrIS is a 1:30am/1:30pm equatorial crossing orbit. The climatology of clouds observed is quite different. 1:30pm is the onset of convection leading to overcast scenes, normally rejected by any retrieval or assimilation scheme. 9:30pm is likely the time for convective cloud detrainment leading to the formation of cirrus anvils. Broken cloud scenes, which are likely to introduce significant scene in-homogeneities, can likely pass the retrival rejection criteria.

• Based on the above consideration we can estimate the effect of the ILS shift to be less important for CrIS than for IASI.

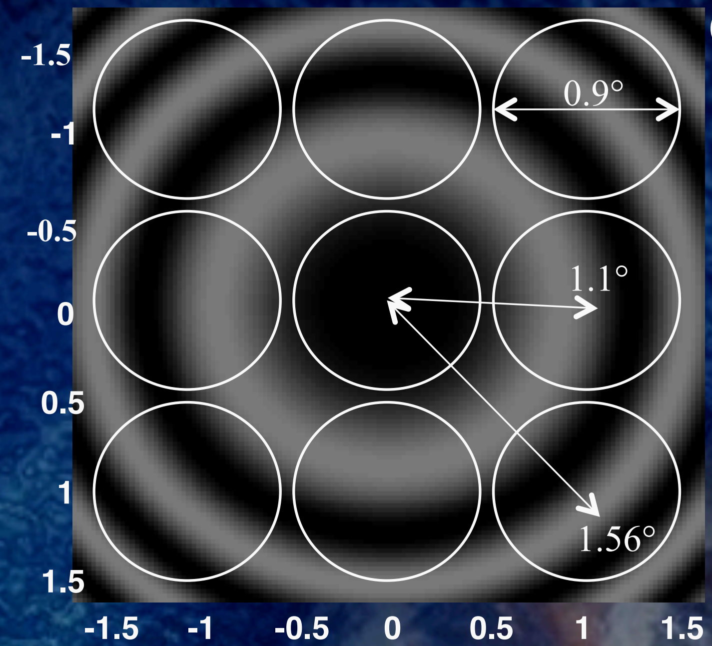
•The only remaining issue to be investigated, though, is CrIS's acquisition geometry. See next slide.

IASI vs CrIS FOV geometry

IASI



CrIS

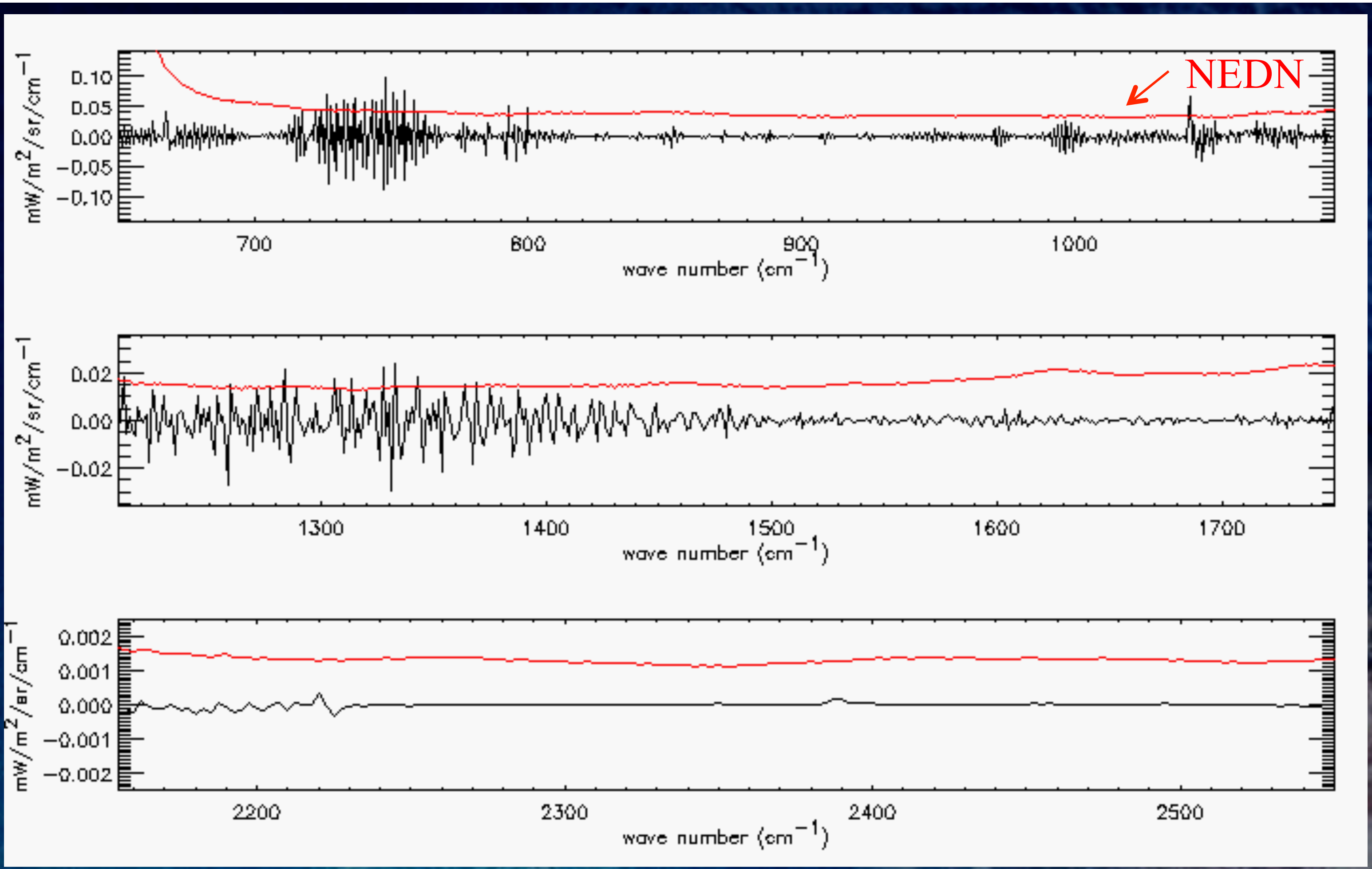


•Applying IASI's δα results to CrIS (assuming sub-pixel heterogeneity and instrument characteristics are close enough between the two instruments):

- •CrIS Side Cube (α=1.1°=0.019rad): $\delta\nu/\nu \sim \alpha\delta\alpha$ = **1.91e-6**
- •CrIS Corner Cube (α=1.56°=0.027rad): $\delta\nu/\nu \sim \alpha\delta\alpha$ = **2.72e-6**

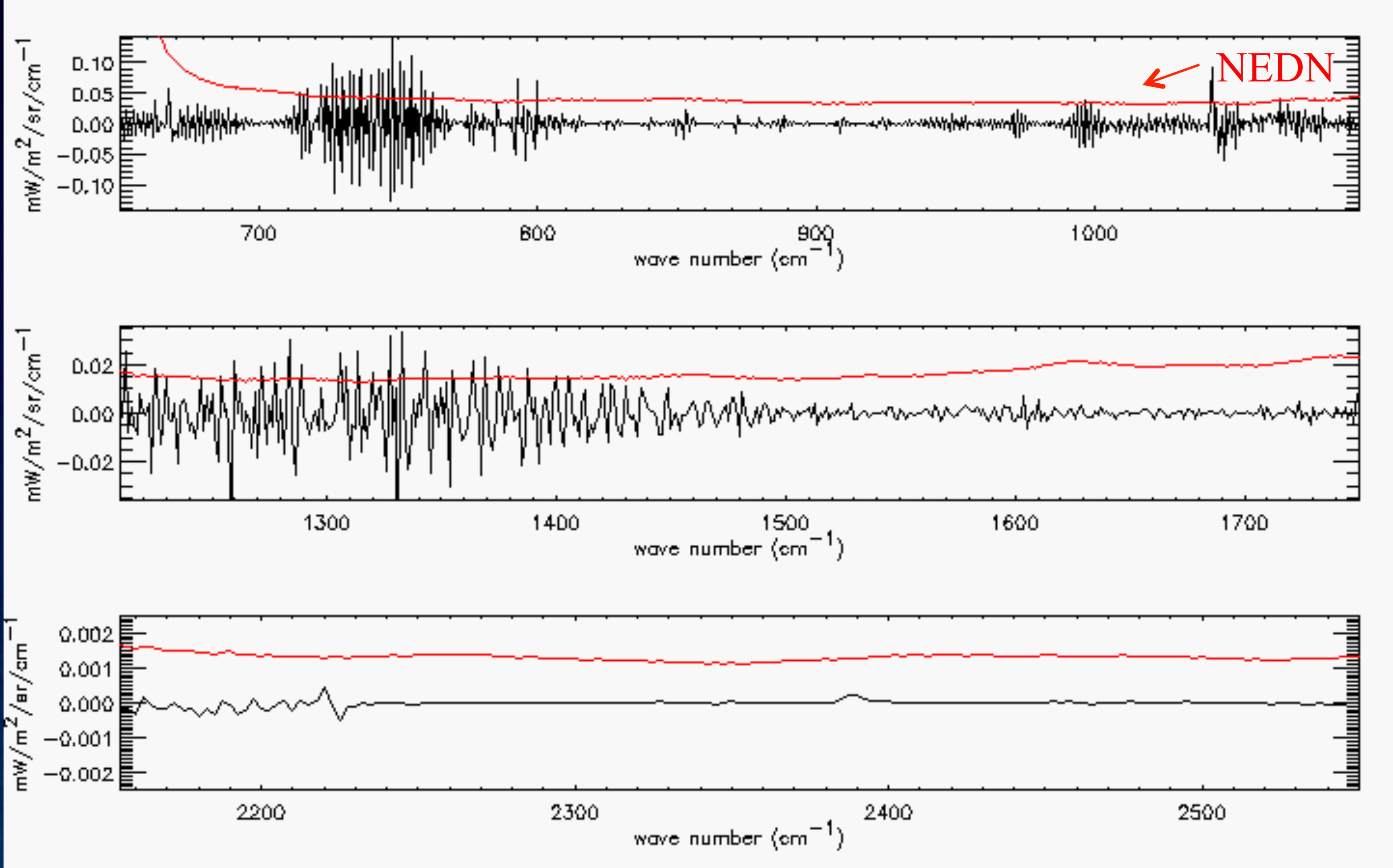
< 3ppm
(spectral calibration requirement)

CrIS Un-apodized radiance error induced by 1 sigma ILS shift - Side cube -



•Radiance error lower than NEDN across the three bands, hence is negligible.

CrIS Un-apodized radiance error induced by 1 sigma ILS shift - Corner cube -



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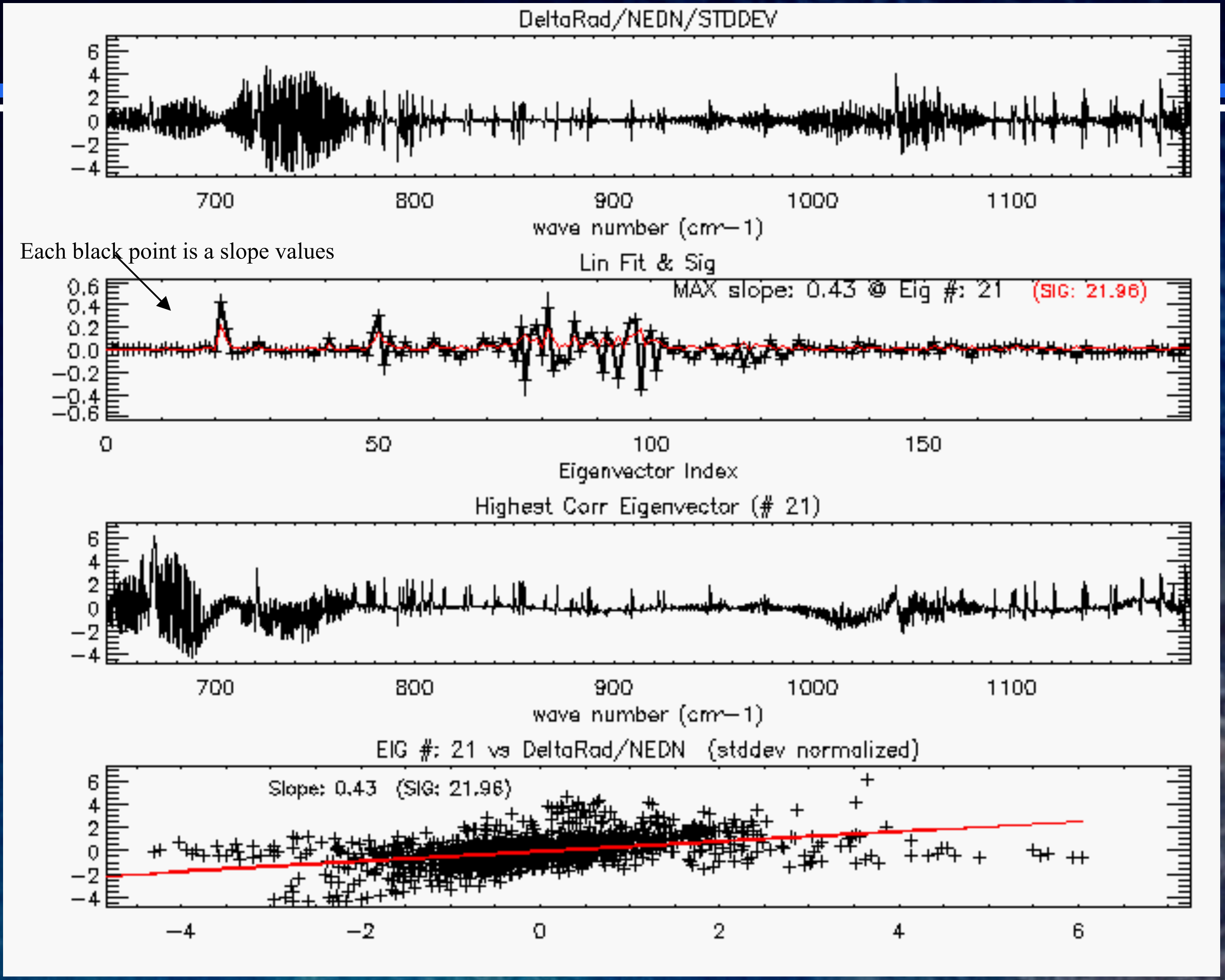
Discussion

- The error introduced by the ILS distortion falls below the instrument noise and is likely to be negligible during retrieval applications with respect to more significant retrieval error sources.

- The problem can become important for radiance-based applications.
- Example: Forcings/Feedbacks studies
  - CO_2 growth rate is 2 ppm/year and introduces a forcing of 0.06K/year at 2388 cm^{-1}
  - AIRS stability < 0.01K/year (radiometric and frequency)allows CO2trends/variability to <0.5 ppm.
  - CrIS frequency errors of 1 ppm = 0.015K at 2388 cm^{-1}
  - Need frequency errors on CrIS <1 ppm to reach AIRS stability and measure 0.5 ppm CO2 forcing.
-

- **Ways to correct for the problem**
- Measure the radiometric displacement and flag these cases out or correct for the problem
  - Need a precise sub-pixel measurement of the radiometric displacement for each frequency.
  - Corrections can be computationally compelling
- Via SVD, reconstruct the radiances removing the eigenvector carrying the ILS shift error
  - Need to identify the correct eigenvector
  - Need to ensure not tossing out important information

Band 1, Gran # 393



BACK UP

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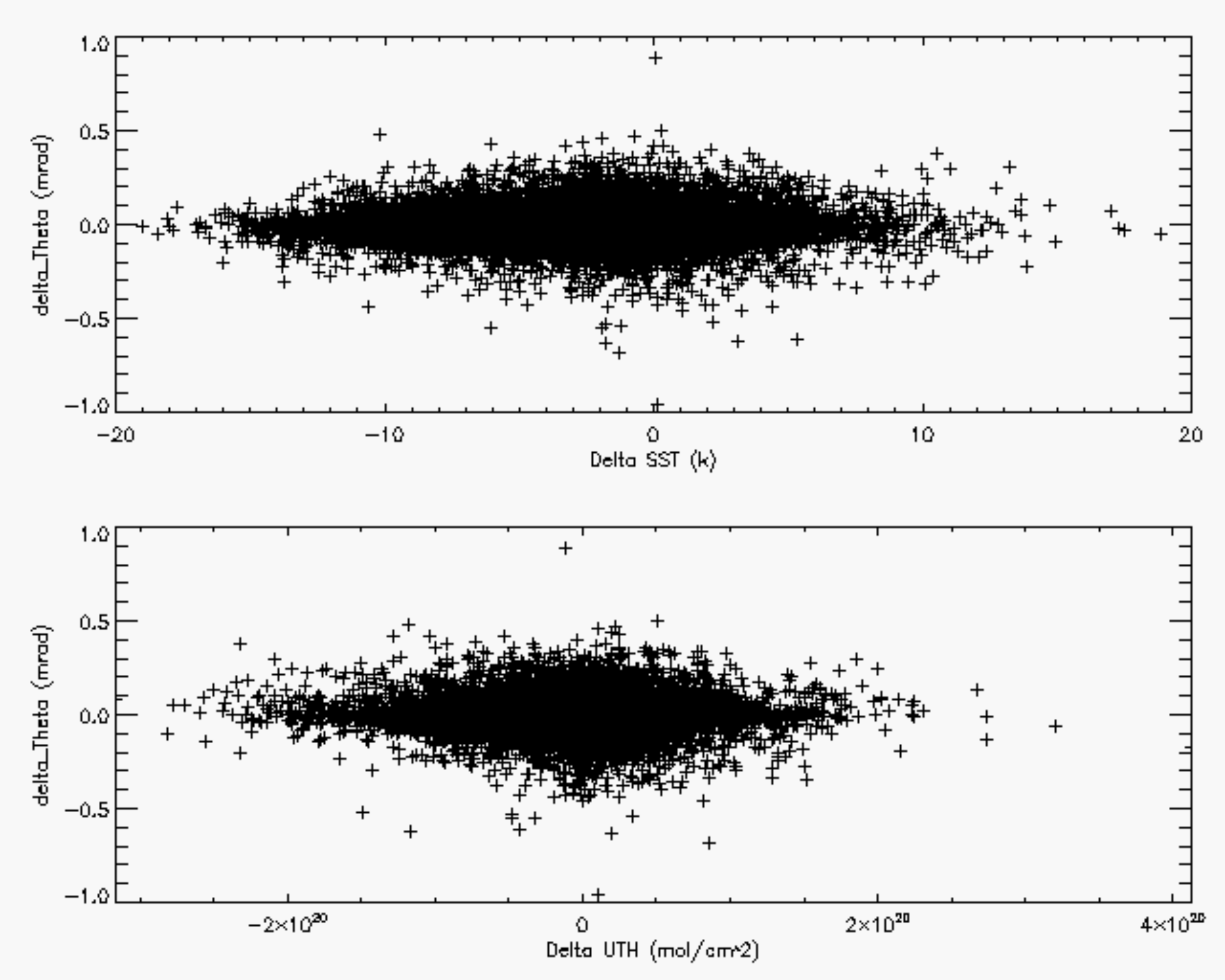

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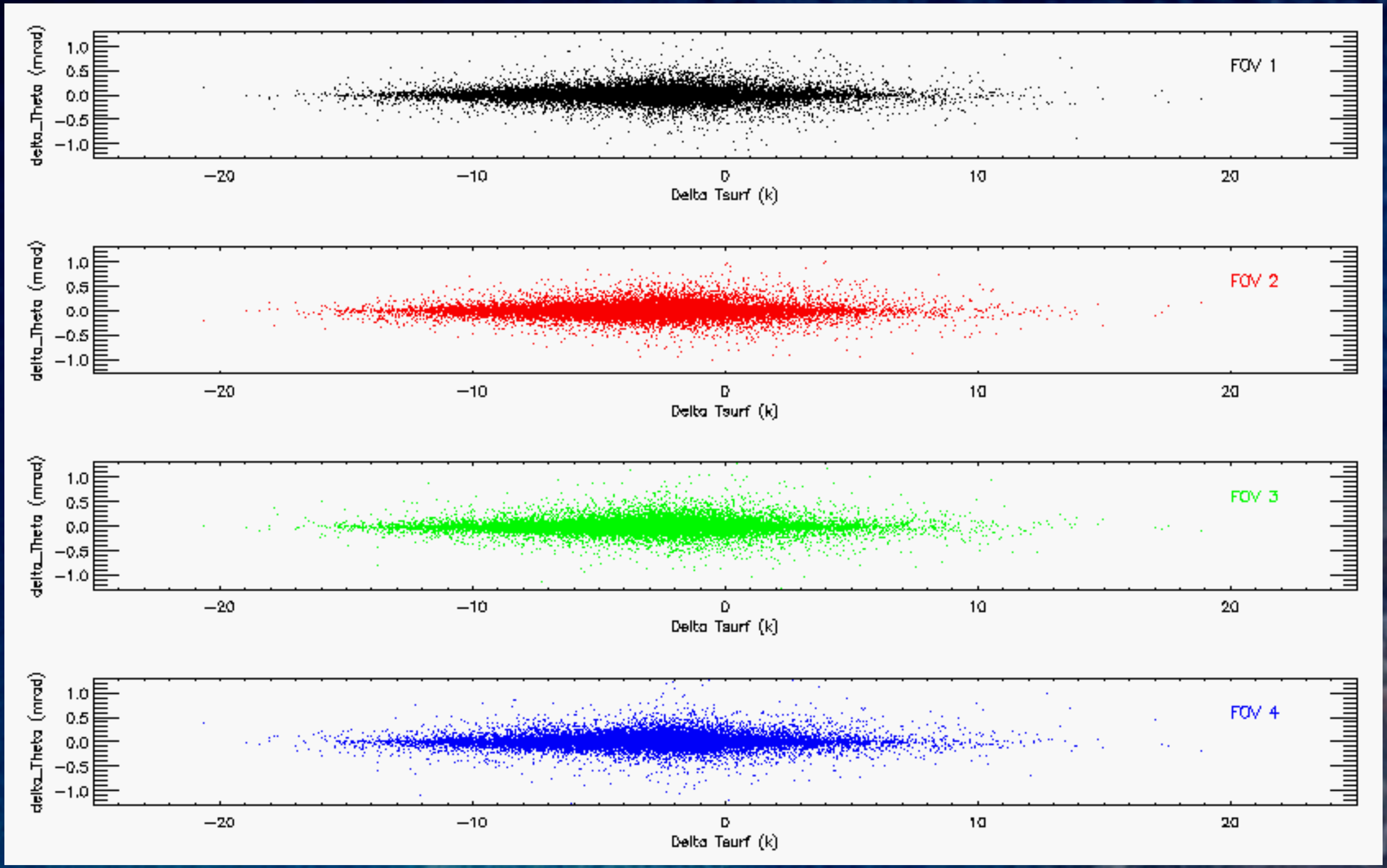
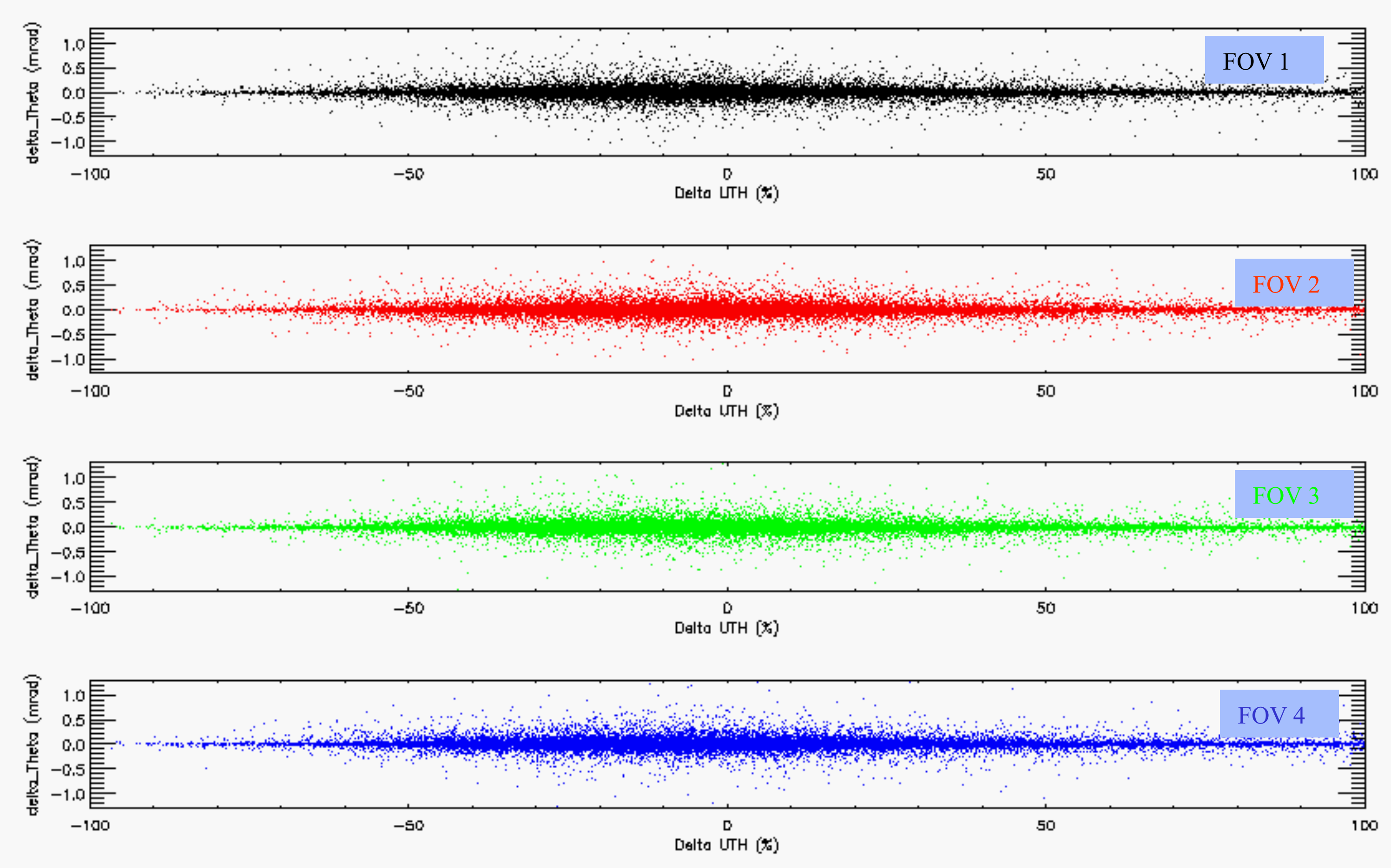


No significant correlation is seen to stand out between each FOV radiometric angular shift and the retrieved SST bias

Centroid Shift vs UTH bias (ret- ecmwf) (Oct 19 2007)



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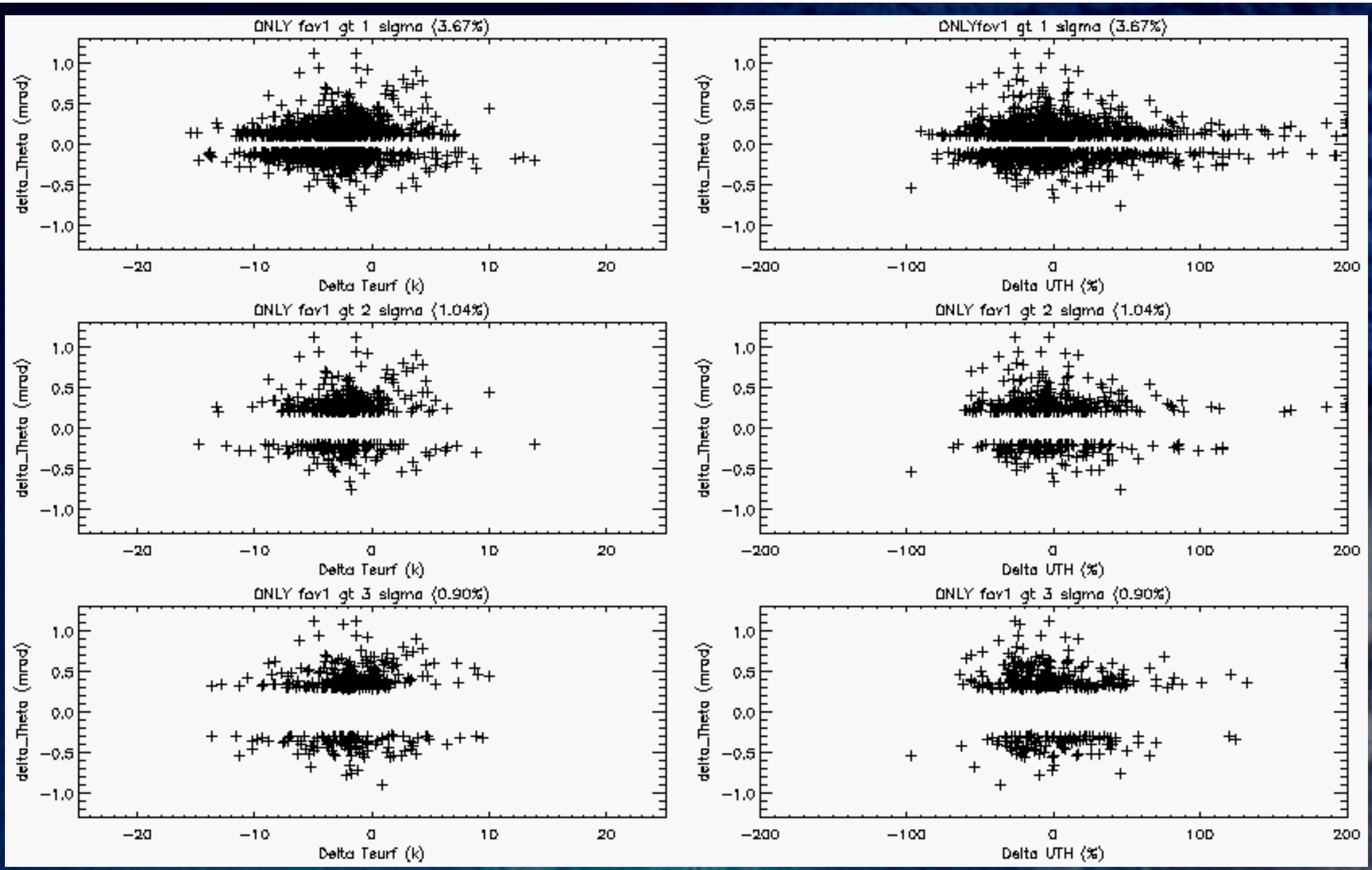

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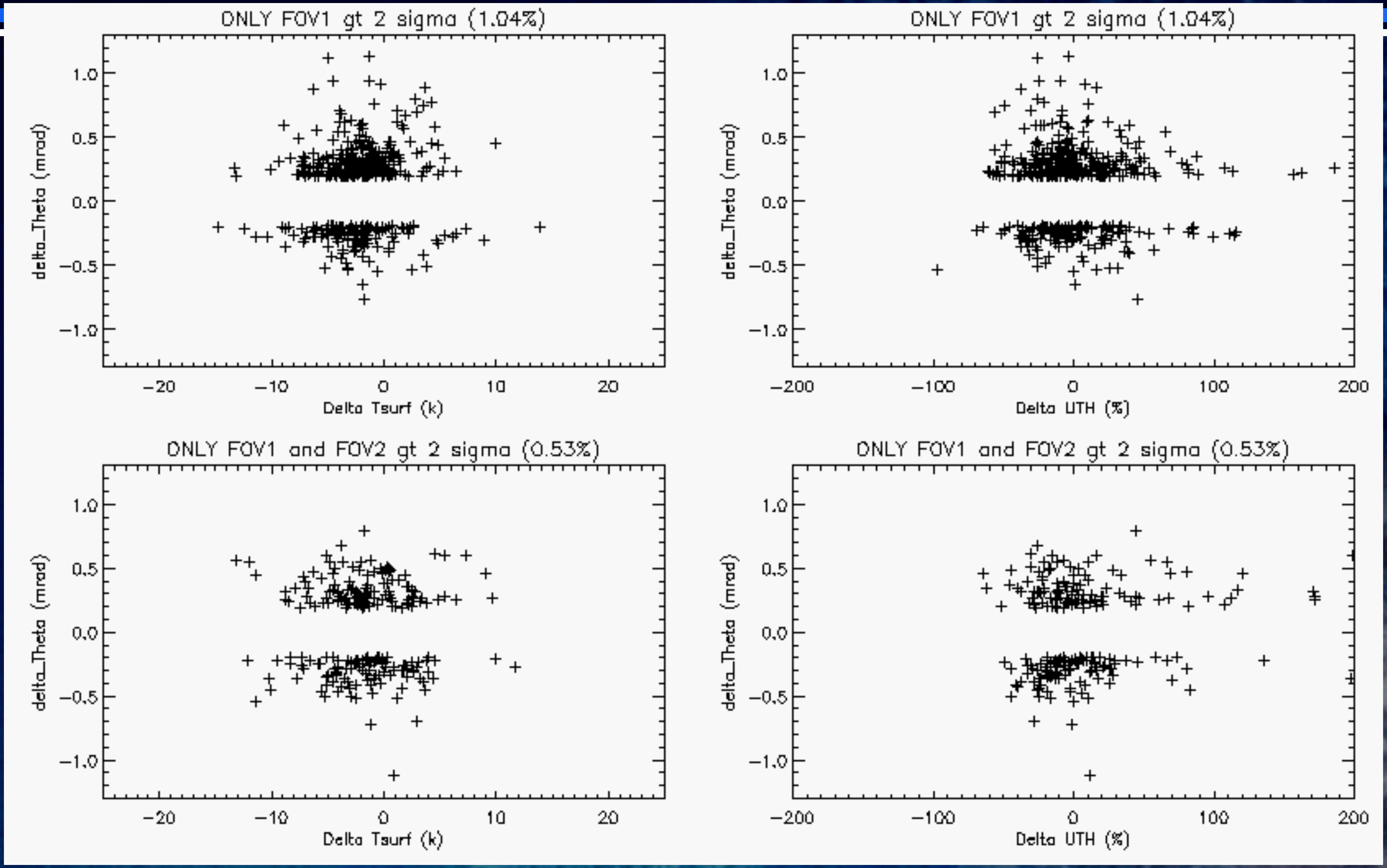

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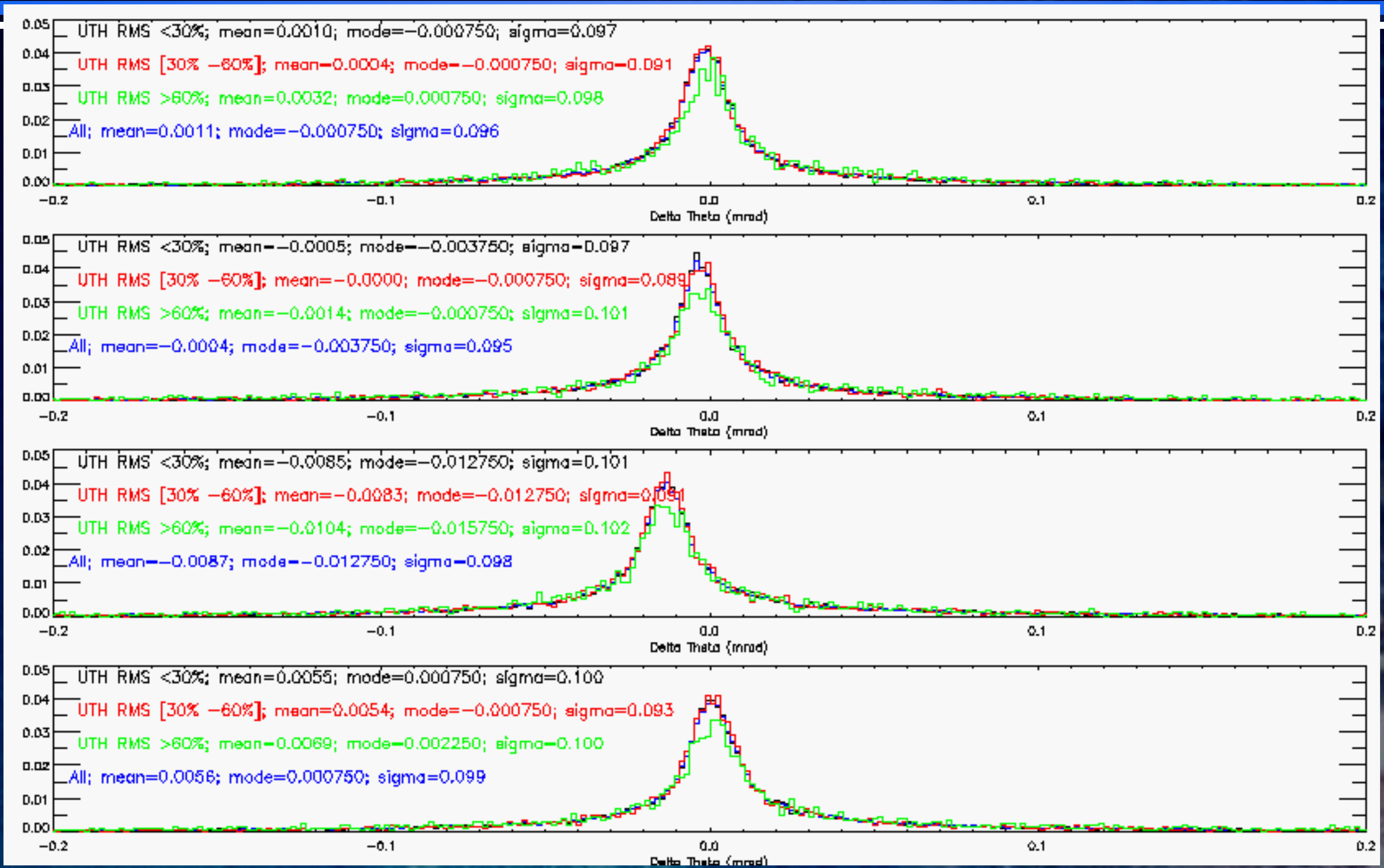

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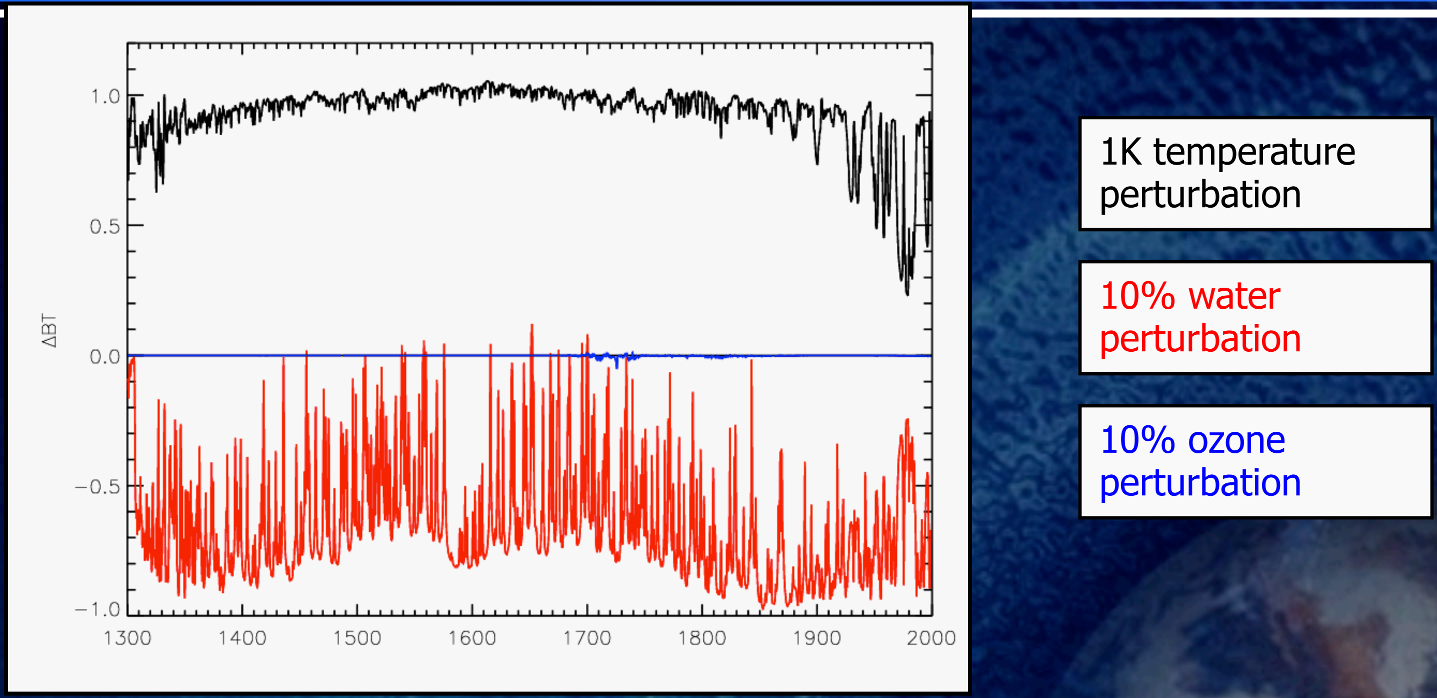

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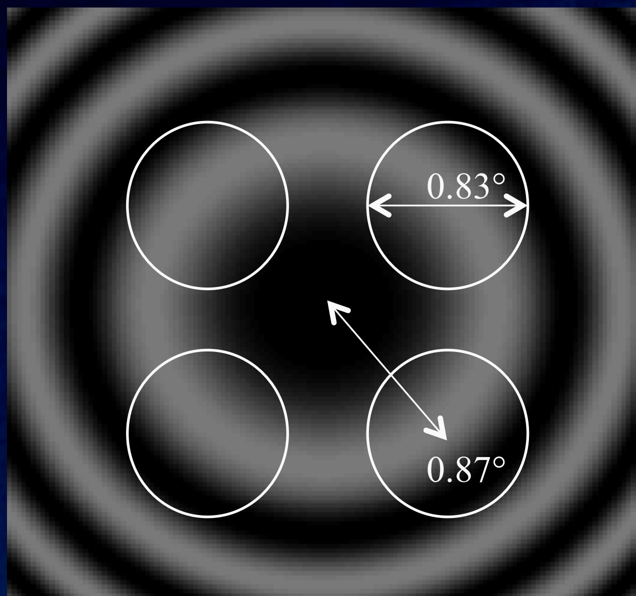
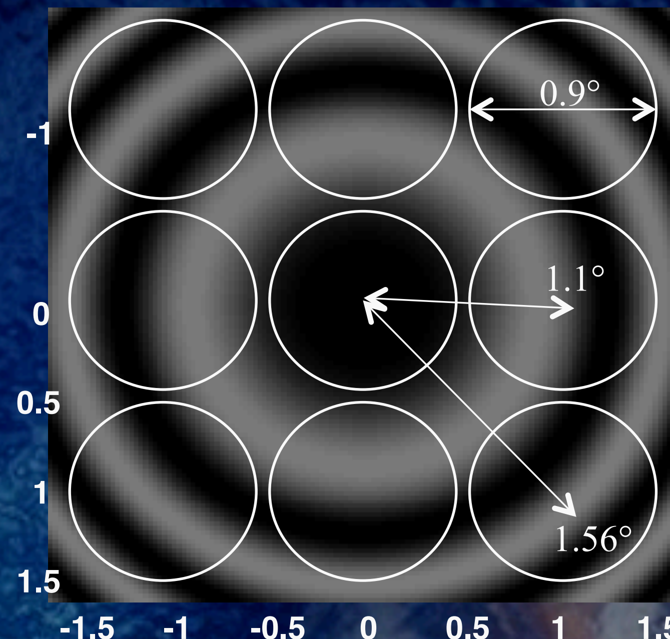


CrIS



- IASI FOVs are arranged in a 2x2 grid on a α=0.87° radius circle from the optical axis.
- CrIS FOVs are arranged on a 3x3 grid on a α=0 °, 1.1 ° and 1.56 ° radius circle from the optical axis.
- For both off-axis CrIS FOVs we should expect a larger frequency shift: $\delta\nu \sim \nu\alpha\delta\alpha$
- Applying IASI's 1sigma angular offset ($\delta\alpha$ =0.1mrad), we should expect, for CrIS, a frequency shift of:
  - CrIS Side Cube (α=1.1°=0.019rad): $\delta\nu/\nu \sim \alpha\delta\alpha$ = 1.91e-6
  - CrIS Corner Cube (α=1.56°=0.027rad): $\delta\nu/\nu \sim \alpha\delta\alpha$ = 2.72e-6
- CrIS ILS frequency shift is of the same order of IASI.
- CrIS ILS frequency shift (1ppm) stable enough to study climate forcings.

Ongoing and Future Work

- Repeat the same analysis done for IASI: perturb CrIS radiance spectrum based on the computed frequency shift and obtain the radiance error in CrIS spectra due to the ILS shift. Assess the impact on retrieval space (Ongoing work).

- What if we find the radiance error to be significant (greater than the instrumental noise)?
- Four approaches:
  - 1) Correct for the ILS shift:
    - Use VIIRS to compute angular offset and correct the self apodization matrix. Computationally very expensive, not feasible for operations.
    - Identify (if it exists) the one eigenvector of the radiance ensemble that correlates the most with the radiance shift. Reconstruct the radiances without that eigenvector. Computationally expensive, high risk methodology (different ensemble have proven to have different highest correlated eigenvector).
  - 2) Identify the cases of significant ILS shift and flag them out.
    - Use PC score analysis to flag anomalous cases. Requires training, might effect the yield considerably.
  - 3) Make the retrieval insensitive to the problem.
    - Incorporate radiance shift signal into the retrieval error covariance matrix to make the retrieval insensitive to this error. Requires training and significant changes to the overall optimization scheme of the retrieval code.
  - 4) Use only the central FOV.
    - Adopt a VIIRS integrated cloud-clearing scheme instead of the 3x3 scheme to perform retrievals. Code already in use for IASI, VIIRS is already collocated to CrIS. Requires testing, limits the spatial resolution of the data.

BACK UP SLIDES

The spectrum of the Source is Given by the Fourier Transform of the Interferogram

In practice the first term is lost due to AC coupling of the detectors.
Interferogram = oscillating part of G(x)

$$I(x) = KY\mathrm{H}\int_0^{\infty} g(\nu)\cos(2\pi\nu \cdot x)d\nu$$

(cm^2 sr) (Volts/Watts) (Watts/cm^2 /sr/cm^{-1}) cm^{-1}= Volts

Detector response (Volts/Watts)
Optical acceptance (cm^2 sr)
Amplifier gain or optical losses

$$I(x) = C\int_{-\infty}^{\infty} g'(\nu)\cos(2\pi\nu x)d\nu = C\int_{-\infty}^{\infty} g'(\nu)\exp(j2\pi\nu x)d\nu$$

$$g'(\nu) = \begin{cases} \dfrac{g(\nu)}{2} & for\, \nu \geq 0 \\ \dfrac{g(\nu)}{2} & for\, \nu < 0 \end{cases}$$

I(x) is the Fourier transform of the source, g(ν).
The spectrum of the source is given back by the Inverse Fourier transform of I(x):

$$g(\nu) = \mathrm{F}^{-1}[I(x)] = \frac{2}{C}\int_{-\infty}^{+\infty} I(x)\exp(j2\pi\nu x)dx$$

Can't measure x over [-∞, ∞]
The interferogram is truncated at L_{max}

Truncation of the Interferogram

The measurement limit is a truncation of the interferogram between $\pm L_{max}$:

$$g_{meas}(\nu) = \frac{2}{C}\int_{-\infty}^{+\infty} A(x) I(x) \exp(j2\pi\nu x)\, dx$$

$$A(x) = \begin{cases} 1; |x| \le L_{\max} \\ 0; |x| > L_{\max} \end{cases}$$

$$g_{meas}(\nu) = \frac{2}{C}\mathrm{F}[A(x)I(x)] = \frac{2}{C}\mathrm{F}[A(x)] \otimes \mathrm{F}[I(x)] = \mathrm{ILS} \otimes \mathrm{g}(\nu)$$

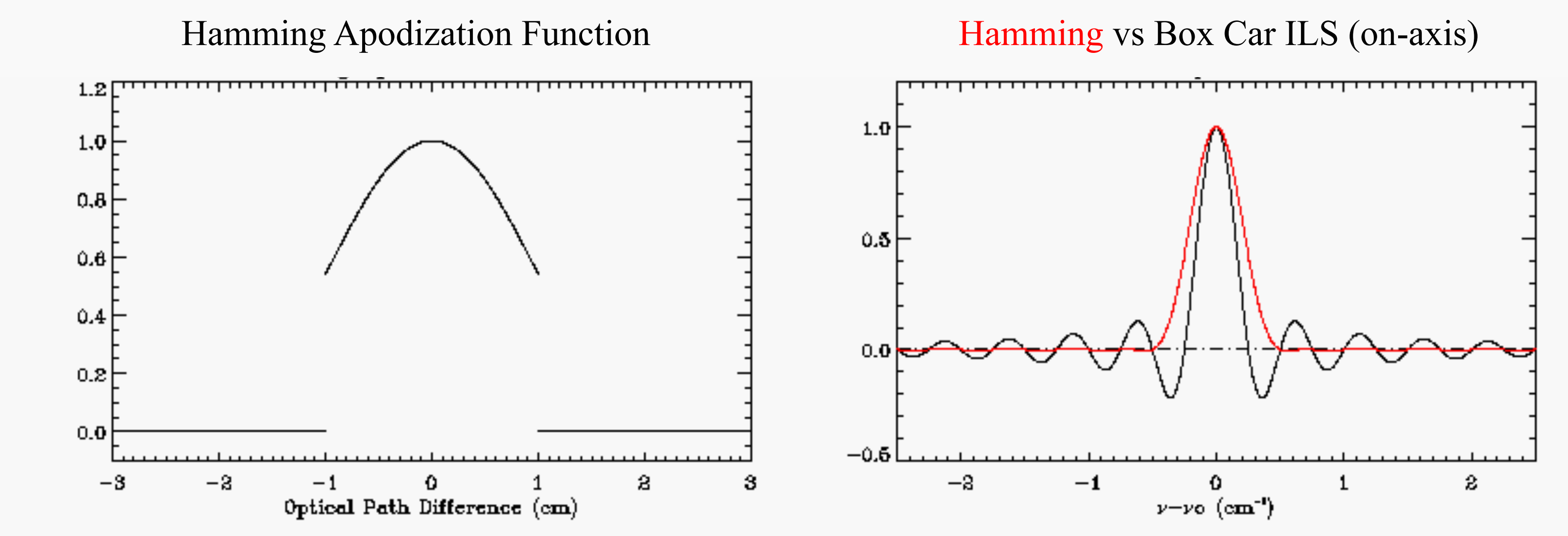
The instrument effect is a loss in accuracy where the original spectrum is "broaden" by the convolution with the instrument line shape function:

$$g_{meas}(\nu) = \int_{-\infty}^{+\infty} ILS(\nu - \nu')\, g(\nu')\, d\nu'$$

In the case of the box car function, A(x):

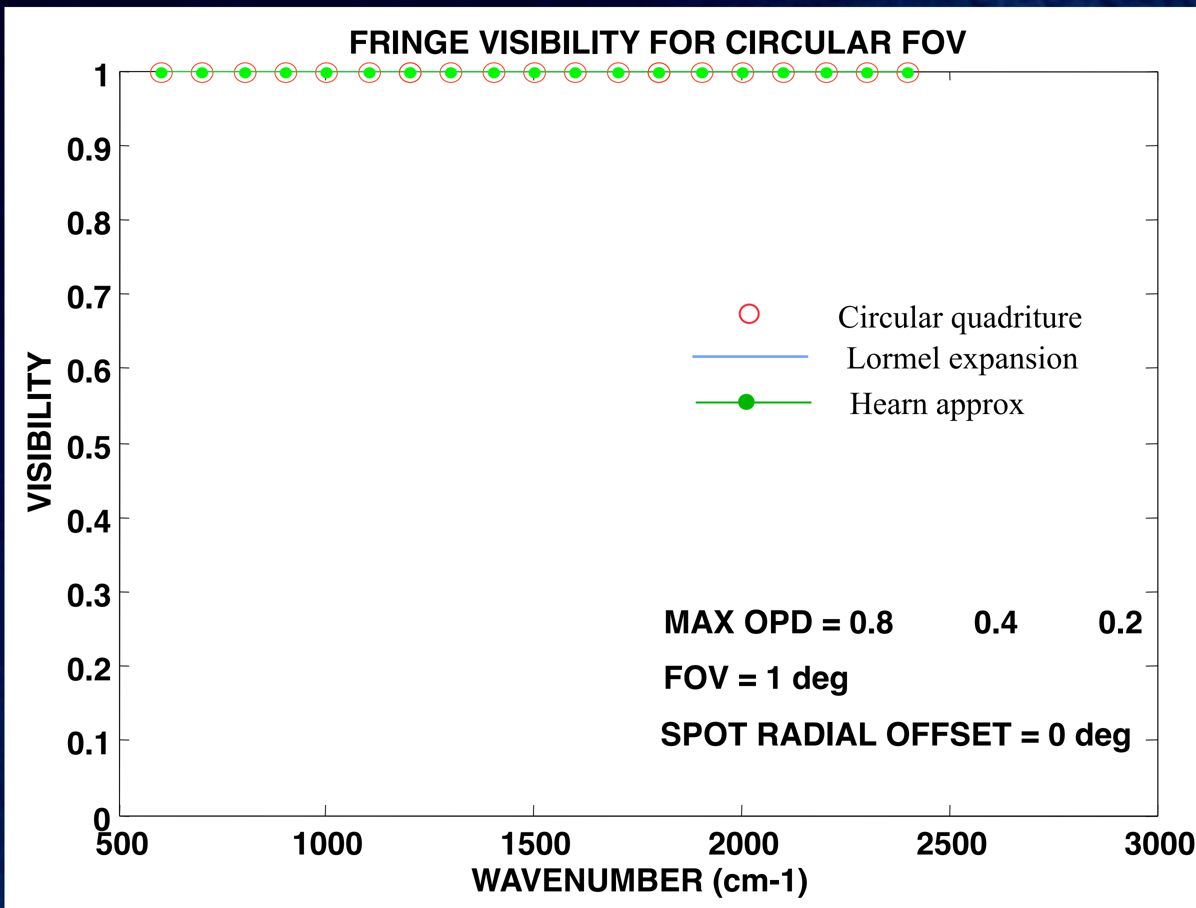
$$ILS = \mathrm{F}[A(x)] = 2L_{\max}\frac{\sin(2\pi\nu L_{\max})}{2\pi\nu L_{\max}} = 2L_{\max}\,\mathrm{sinc}(2\pi\nu L_{\max})$$

Hamming Apodization Function (on-axis monochromatic input)

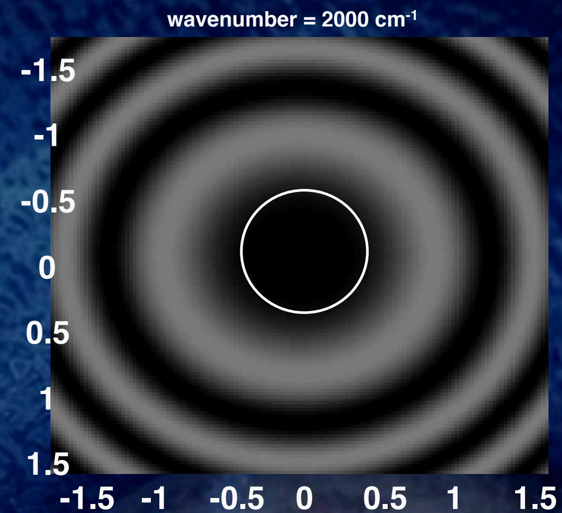


The Instrument Line Shape resulting from the Hamming truncation function is a more smoothed function that gets rid of side lobe effects with the penalty of lower spectral resolution and correlated adjacent channels.

Visibility for CrIS Central FOV



Picture courtesy of D. Mooney

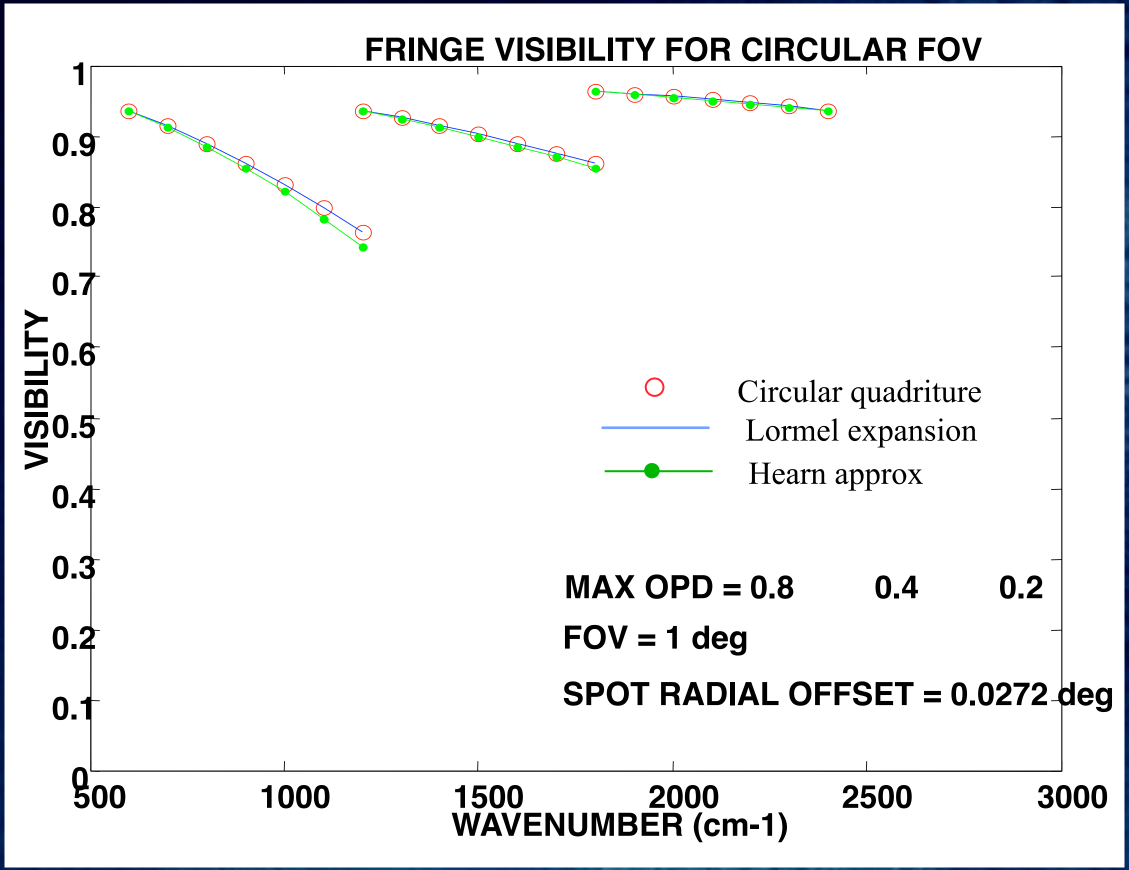


Define visibility V(x):

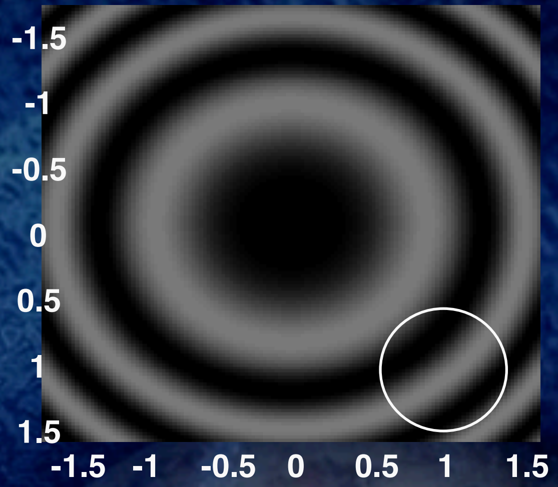
$$V(X)=\frac{G_{\max}(x)-G_{\min}(x)}{G_{\max}(x)+G_{\min}(x)}$$

The central detector falls inside the central bright spot. There is no loss in signal, V(x) =1

Visibility for CrIS Corner FOV



Picture courtesy of D. Mooney



Define visibility V(x):

$$V(X) = \frac{G_{\max}(x) - G_{\min}(x)}{G_{\max}(x) + G_{\min}(x)}$$

The corner detector falls outside the central bright spot. There is a loss in signal, V(x) <1.

Self Apodization Matrix

Polychromatic, homogeneous source:

$$g_{meas}(v) = \sum_{FOV} ILS_{x,y}(v - v_0) \otimes g(v_0) + \sum_{FOV} ILS_{x,y}(v - v_1) \otimes g(v_1) + \ldots + \sum_{FOV} ILS_{x,y}(v - v_n) \otimes g(v_n)$$

$$g_{meas}(v) = \mathrm{ILS}_{FOV}(v - v_0) \otimes g(v_0) + \mathrm{ILS}_{FOV}(v - v_1) \otimes g(v_1) + \ldots$$

In matrix form ("Self Apodization Matrix"):

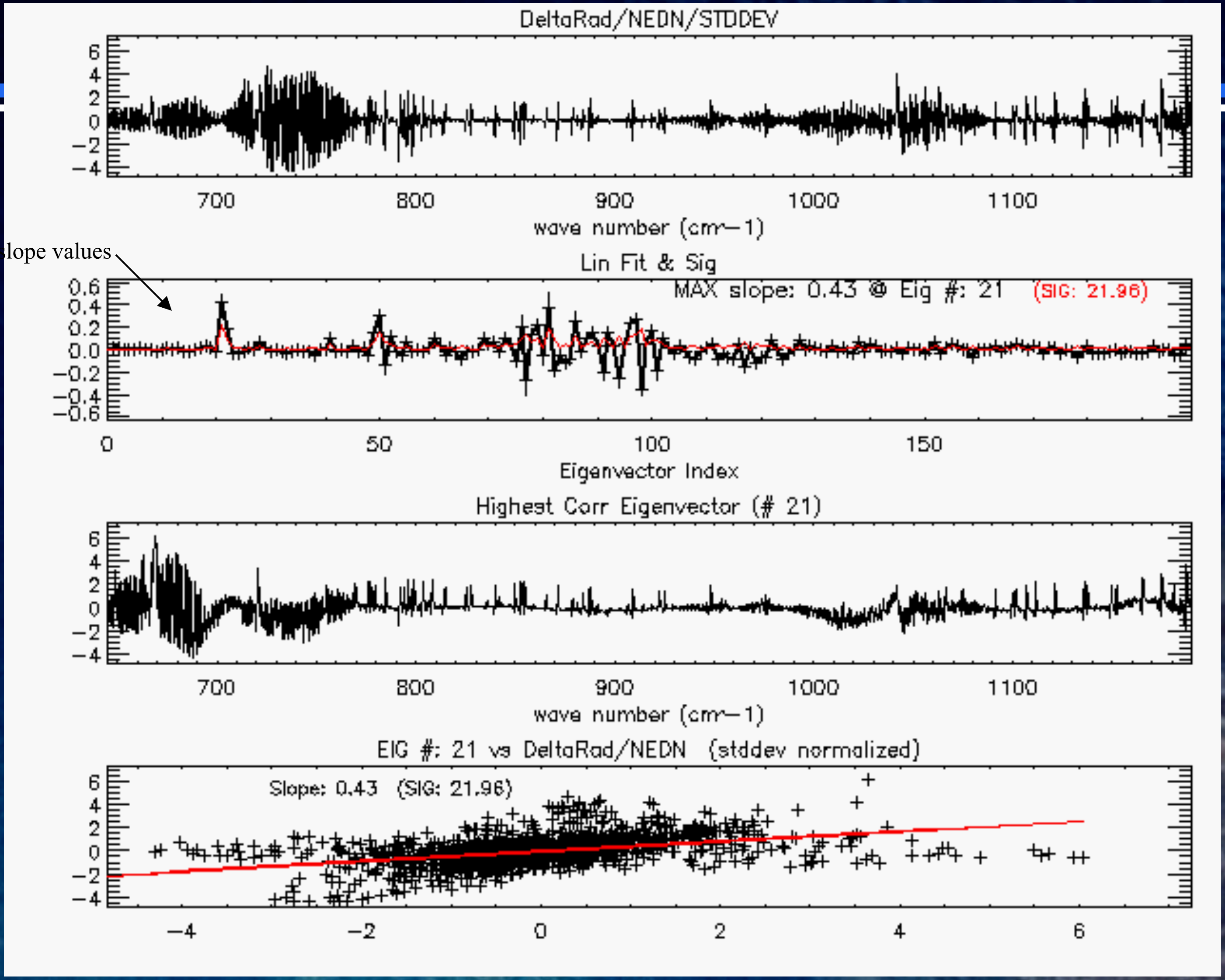
$$\begin{bmatrix} g_{meas}(\nu_0) \\ g_{meas}(\nu_1) \\ \cdots \\ \cdots \\ \cdots \\ g_{meas}(\nu_n) \end{bmatrix} = \begin{bmatrix} \mathrm{ILS_{FOV}}(\nu_0 - \nu_0) & \mathrm{ILS_{FOV}}(\nu_0 - \nu_1) & \ldots & \mathrm{ILS_{FOV}}(\nu_0 - \nu_n) \\ \mathrm{ILS_{FOV}}(\nu_1 - \nu_0) & \mathrm{ILS_{FOV}}(\nu_1 - \nu_1) & \ldots & \mathrm{ILS_{FOV}}(\nu_1 - \nu_n) \\ & \cdots & & \\ & \cdots & & \\ & & & \\ \mathrm{ILS_{FOV}}(\nu_n - \nu_0) & \mathrm{ILS_{FOV}}(\nu_n - \nu_1) & \ldots & \mathrm{ILS_{FOV}}(\nu_n - \nu_n) \end{bmatrix} \begin{bmatrix} g_{\alpha=0}(\nu_0) \\ g_{\alpha=0}(\nu_1) \\ \cdots \\ \cdots \\ \\ g_{\alpha=0}(\nu_n) \end{bmatrix}$$

IMPORTANT: The inversion of the self apodization matrix allows for off-axis correction and removes the self-apodization effect of the 9 FOVs.

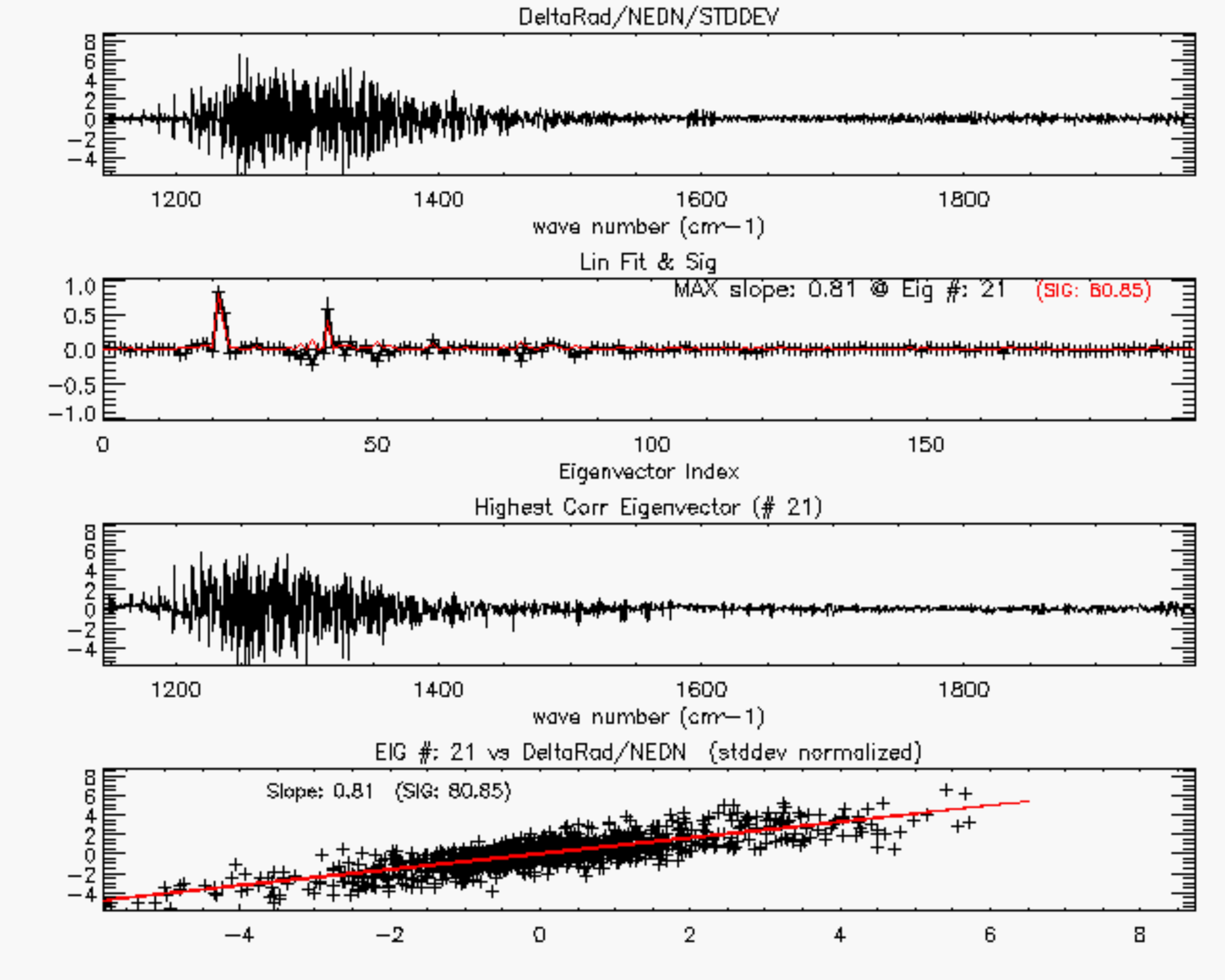
The self apodization removal requires accurate knowledge of each $\mathrm{ILS_{FOV}}$ ("Nominal ILS")

Band 1, Gran # 393

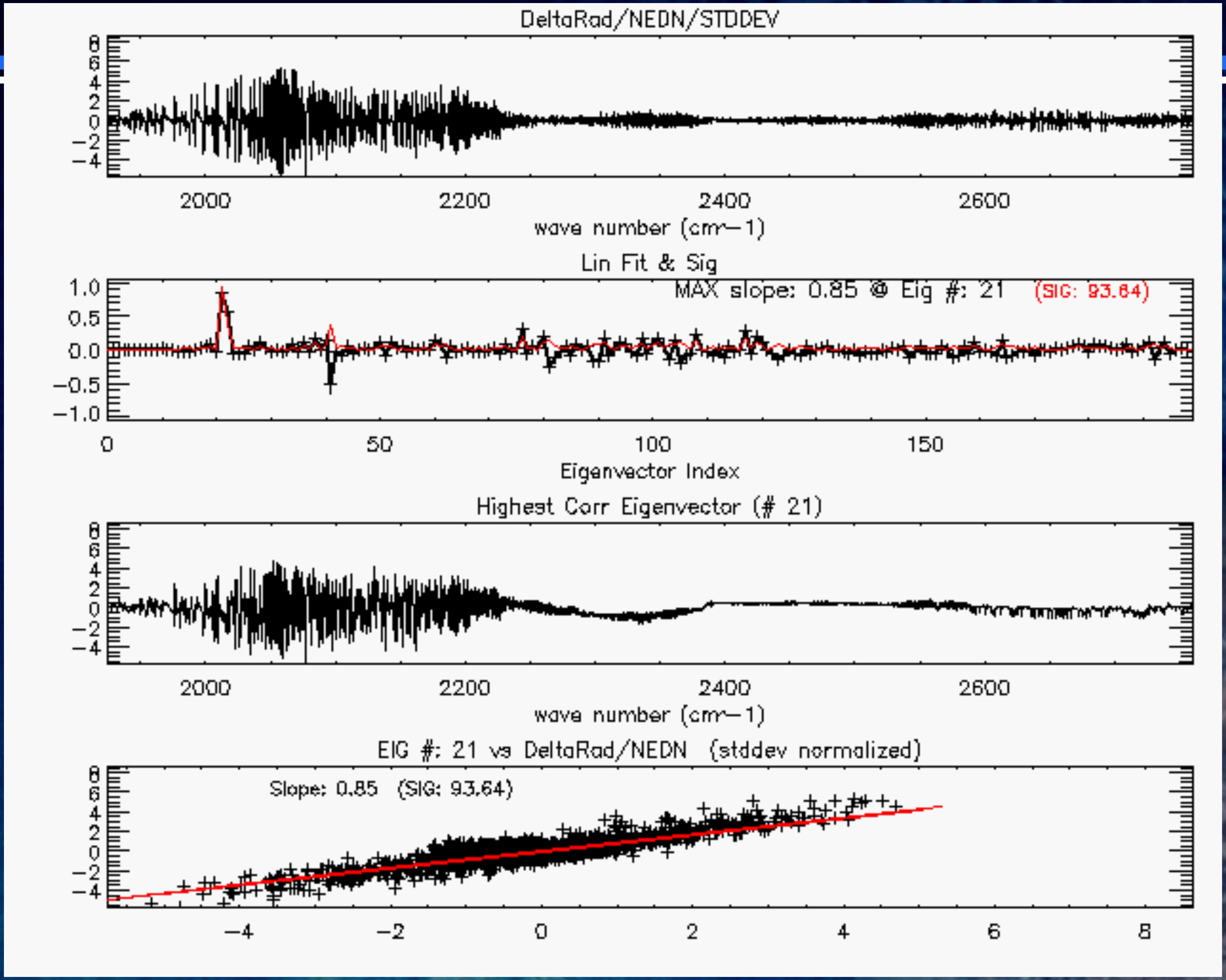
Each black point is a slope values



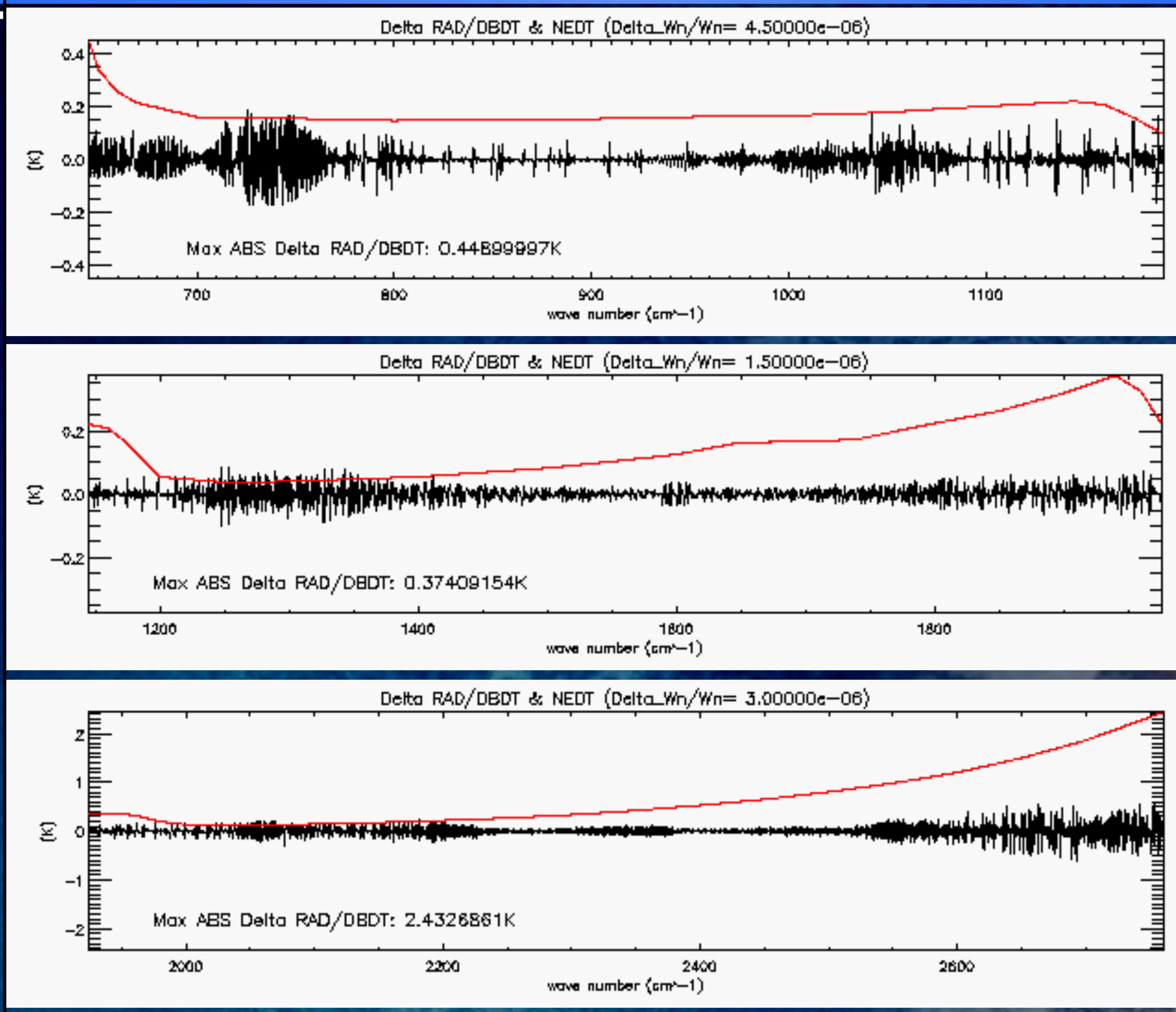
Band 2, Gran # 393



Band 3, Gran # 393

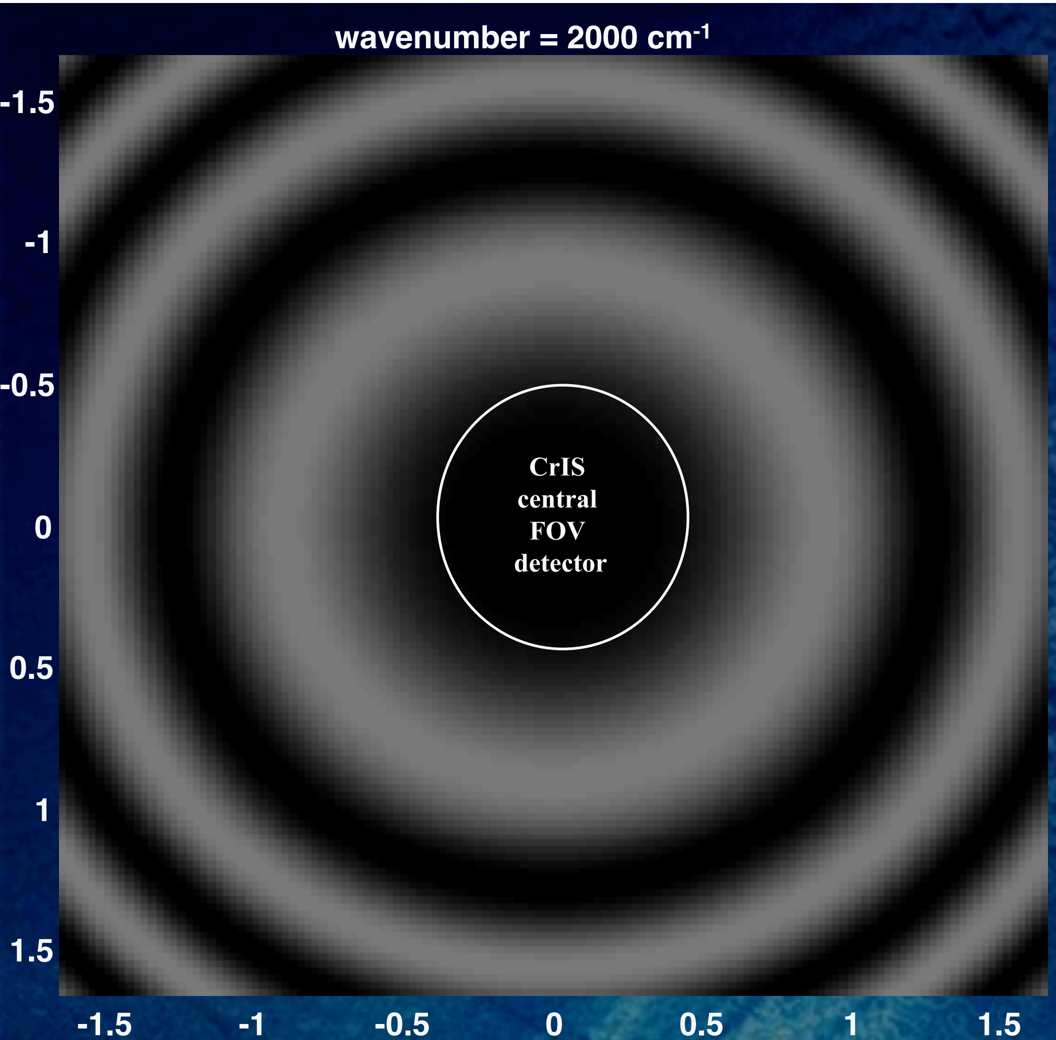


Band 1, 2, and 3 BT error summary



Self Apodization Effect & CrIS FOV geometry

$$\frac{\nu}{\Delta\nu} = 2x_{max}\nu$$



What the detector measures is the integration over the solid angle subtended by the detector at the exit pupil:

$$G(x) = \frac{g(\nu)}{\Omega_{max}} \int_0^{\Omega_{max}} [1 + \cos 2\pi\nu x(1 - \frac{\Omega'}{2\pi})] d\Omega'$$

$$ILS(\nu) = \mathrm{F}[\mathrm{G(x)}] \propto \mathrm{sinc}(2\pi\nu x_{max})$$

Spectral Resolution: (RSR): $\Delta\nu = \frac{1}{2x_{max}}$

Visibility: $V(X) = \frac{G_{max}(x) - G_{min}(x)}{G_{max}(x) + G_{min}(x)}$

Spectral Resolution – Visibility Trade off

X ↑ **SR** ↑ **V** ↓